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Postbuckling behavior of circular higher-order shear deformable nanoplates including surface energy effects

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ABSTRACT

The effects of surface energy are generally ignored in traditional continuum elasticity. However, due to the high surface to volume ratio in nanostructures, this is not the case for them. In this work, the nonlinear postbuckling characteristics of circular nanoplates are predicted in the presence of surface energy effects including surface elasticity and residual surface tension. For this objective, Gurtin–Murdoch elasticity theory is implemented into the classical higher-order shear deformation plate theory. In order to satisfy the balance conditions on the surfaces of nanoplate, it is assumed the normal stress of the bulk is distributed cubically through the thickness of nanoplate. Virtual work's principle in conjunction with von Karman geometric nonlinearity is utilized to derive non-classical nonlinear governing differential equations of motion and related boundary conditions. Afterwards, an efficient numerical methodology based generalized differential quadrature (GDQ) method is carried out using the shifted Chebyshev–Gauss–Lobatto grid points to discretize the governing partial differential equations. Then, the Galerkin's method is employed to reduce the set of nonlinear equations into a time-varying set of ordinary differential equations of Duffing type. At the end, the pseudo arc-length continuation technique is utilized in order to obtain the solution of the parameterized equation.
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1. Introduction

The recent developments in science and technology cause to open new era in fabrication and manufacturing that leads to a significant role of the structures at nanoscale in many engineering fields. Among these nanostructures, nanoplates have received much attention and have become a subject of extensive research due to their widespread applications in nano-electro-mechanical-systems (NEMS). In these applications, nanostructure-dependent size effects are often considerable. To capture such small scale effects on the mechanical responses, modified continuum elasticity theories have been introduced and applied in various researches [1–17].

Surface energy effect is one of size effect which has been shown by the experiment of Chen et al. [18]. If the characteristic length of a material element is comparable to the intrinsic length, the surface energy effects play an important role in the mechanical responses and some size-dependent behavior can be observed. Hence, surface energy effects for macroscopic materials are negligible compared to bulk energy. However, because of high surface to volume ratio in nanostructures, surface energy effects should take into account in order to obtain more accurate results at this sub-micron size. On the basis

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of a rigorous mathematical formulation, Gurtin and Murdoch [19,20] developed a mathematical framework based on continuum elasticity in which the surface can actually be treated as an elastic layer with zero thickness perfectly bonded to the bulk. By using Gurtin–Murdoch elasticity theory, several attempts have been carried out to predict the surface energy effects on the mechanical characteristics of nanostructures.

He et al. [21] obtained an analytical model to study the size-dependent mechanical response of ultra-thin elastic films based on Gurtin–Murdoch elasticity theory. Wang and Feng [22] discussed the influences of surface elasticity and residual surface stress on the natural frequency of nanowires. They indicated that both the surface elastic modulus and residual surface stress contribute to the vibration response of nanowires. He and Lilley [23,24] studied the elastic responses of static and dynamic bending of nanowires with various boundary conditions based on surface elasticity theory. Huang [25] developed a modified continuum model of elastic films with nanoscale thickness by incorporating surface elasticity into the conventional nonlinear von Karman plate theory. Park [26] indicated that the influence of residual surface stress on the frequency of silicon nanowires is considerable for finite deformation kinematics. Lui and Rajapakse [27] proposed an analytical solution based on Gurtin–Murdoch elasticity theory for the static response and free vibration of thin and thick nanobeams corresponding to different loading conditions and end supports. Ansari and Sahmani [28] predicted the bending and buckling behaviors of nanobeams including surface stress effect by employing Gurtin–Murdoch elasticity theory within the frameworks of Euler–Bernoulli, Timoshenko, Reddy and Levinson beam theories. Ansari and Sahmani [29] examined the surface effects on the free vibration characteristics of nanoplates using surface elasticity theory.

Recently, Sharabiani and Yazdi [30] investigated nonlinear free vibration of functionally graded nanobeams including surface effects. Ansari et al. [31,32] studied postbuckling response of Euler–Bernoulli and Timoshenko nanobeams incorporating the surface effects on the basis of Gurtin–Murdoch elasticity theory. Peng and Huang [33] performed vibration frequency analysis of cylindrical nanotubes with the effect of surface stress and surface inertia. They used surface elasticity theory to calculate the phonon dispersion and the resonant frequencies for the specific vibration modes. Zhang et al. [34] obtained two-dimensional equations of piezoelectric plates with nano-thickness which take account of the surface effects. They considered the piezoelectric plate as a bulk core plus two surface layers. Wang and Wang [35] studied the influence of surface free energy on the postbuckling behavior of Kirchhoff rectangular nanoplates. Ansari et al. [36] implemented Gurtin–Murdoch elasticity theory in conjunction with von Karman nonlinear geometrically into the classical Timoshenko beam theory to analyze nonlinear forced vibration response of nanobeams with surface effects. They also investigated the free vibration response of postbuckled circular Mindlin nanoplate based on surface elasticity theory [37]. In a recent work, Sahmani et al. [38] predicted natural frequencies of third-order shear deformable nanobeams in the vicinity of the postbuckling domain incorporating surface effects.

The objective of the present study is to predict the nonlinear postbuckling characteristics of circular higher-order shear deformable nanoplates with consideration of surface effects. For this purpose, Gurtin–Murdoch elasticity theory is implemented into the classical higher-order shear deformation plate theory to develop non-classical plate model which has an excellent capability to capture the effects of surface energy. On the basis of variational approach, size-dependent nonlinear governing differential equations of motion and associated boundary conditions are derived. Subsequently, an efficient numerical methodology is performed to obtain nonlinear equilibrium path of postbuckling response. To this end, generalized differential quadrature (GDQ) method is carried out using the shifted Chebyshev–Gauss–Lobatto grid points to discretize the governing partial differential equations. Then, the Galerkin's technique is employed to reduce the set of nonlinear equations into a time-varying set of ordinary differential equations of Duffing type.

2. Preliminaries

Fig. 1 shows schematically a uniform circular nanoplate with radius R, and thickness h. A coordinate system r, θ, z is attached to the center of nanoplate as the z axis is taken along the thickness. In the present work, higher-order shear deformation plate theory is utilized. This theory assumes that the transverse shear strains are changed parabolically through the plate thickness. The advantage of this theory over other types of plate theory is that no shear correction factor is required. Therefore, the displacement components u_r, u_θ, u_z along the axes r, θ, z, respectively, are considered as

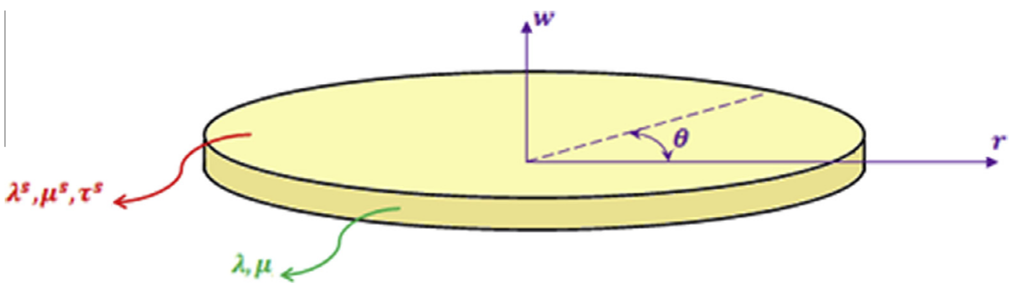

Fig. 1. Schematic of a circular nanoplate carrying surface effects: kinematic parameters, coordinate system and geometry.

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