Applied Mathematical Modelling 39 (2015) 3704–3720

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Analytical study on the transient heating of a two-dimensional skin tissue using parabolic and hyperbolic bioheat transfer equations

Hossein Askarizadeh [1], Hossein Ahmadikia *

Department of Mechanical Engineering, University of Isfahan, 81746-73441 Isfahan, Iran

ARTICLE INFO

Article history:
Received 30 March 2014
Received in revised form 21 September 2014
Accepted 1 December 2014
Available online 12 December 2014

Keywords:
Skin tissue
Two-dimensional
Bioheat transfer
Non-Fourier
Dual-phase-lag
Analytical solution

ABSTRACT

In this study, exact analytical analysis of two-dimensional Fourier and non-Fourier bioheat transfer equations in skin tissue exposed to an instantaneous heating condition is presented. The effects of blood perfusion and metabolic heat generation on the tissue thermal behavior are considered. Corresponding analytical approach is developed through Laplace transform (LT) technique in conjunction with the separation of variables method and inversion theorem. The dual-phase-lag (DPL), thermal wave (TW) and Pennes models of bioheat transfer equation are studied by utilizing a generalized model. The reliability of the presented results has been evidenced through enforcing appropriate circumstances on the DPL model and comparing the outcome results with that of predicted by the Pennes and TW models. It is proved that the DPL bioheat transfer equation with the effects of blood perfusion and metabolic heat generation can be reduced to the Pennes bioheat transfer equation when $\tau_q = \tau_T$. The effects of local non-equilibrium on the tissue thermal behavior are examined and discussed by comparing the gradient precedence (GP) and flux precedence (FP) heat flow regimes of the DPL model. The first- and second-degree burn times of a 2D skin tissue for different bioheat models are introduced and compared with the 1D case.

1. Introduction

Technology advances lead to modern thermotherapeutic methods which involve different techniques such as laser, radio frequency, focused ultrasound, and microwaves for heating biological tissues in a safe manner. For example, the laser beam is focused to the tumor site by an objective lens for thermal therapy. One of the challenges in thermal therapy methods is delivering the appropriate volume of heat to the specified section of the patient's body. In order to have a successful thermotherapeutic operation, pre-treatment planning, image-guided surgical control, and post-treatment monitoring are necessary. The objective of this procedure is to predict the tissue temperature field and identify the damaged tissue regions as accurate and appropriate as possible [1]. Thus accurate prediction of temperature distributions, damaged regions and heat transfer rate in biological tissues during the thermal treatment processes is important both for treatment planning and for designing new clinical heating systems.

* Corresponding author. Tel.: +98 (313)7932965; fax: +98 (313)7932746.
E-mail addresses: H.Askarizadeh@ymail.com (H. Askarizadeh), ahmadikia@eng.ui.ac.ir (H. Ahmadikia).
[1] Tel.: +98 (313)7934517; fax: +98 (313)7932746.

http://dx.doi.org/10.1016/j.apm.2014.12.003

Nomenclature

a_1, a_2	constant coefficients, Eq. (18b)
A	constant coefficient, Eq. (9)
A_0	frequency factor (s^{-1})
B	constant coefficient, Eq. (9)
c	tissue specific heat ($J\,kg^{-1}\,K^{-1}$)
c_b	blood specific heat ($J\,kg^{-1}\,K^{-1}$)
C	thermal wave speed ($m\,s^{-1}$)
D	constant coefficient, Eq. (9)
e_n	constant coefficients, Eq. (19b)
E	dimensionless temperature function employed in separation of variables method
E_a	activation energy of denaturation reaction ($J\,mol^{-1}$)
E_n	particular functions, Eq. (19b)
F_{mn}	square of λ_n plus square of ratio of υ_m to ξ_L, $\lambda_n^2 + (\upsilon_m/\xi_L)^2$
i	imaginary unit
I_0, I_1	modified Bessel functions of the first kind
j	non-negative integer number
J_0, J_1	Bessel functions of the first kind
k	tissue thermal conductivity ($W\,m^{-1}\,K^{-1}$)
K_0	modified Bessel function of the second kind
l	infinite line employed in Eq. (23)
L	tissue length, m
m	non-negative integer number
m_1, m_2	roots of the characteristic equation of homogeneous part of the ordinary differential equation (12b)
M_n	constant coefficients, Eq. (21a)
M_0	constant coefficient, Eq. (21b)
n	non-negative integer number
$\boldsymbol{q}$	heat flux vector ($W\,m^{-2}$)
q_0	incident heat flux intensity ($W\,m^{-2}$)
Q_{met}	metabolic heat generation ($W\,m^{-3}$)
r	radial coordinate (m)
R	universal gas constant ($J\,mol^{-1}\,K^{-1}$)
R_p	incident heat flux spot radius (m)
R_t	tissue radius (m)
s	Laplace domain parameter
S_{mnj}	singular points of $\overline{\Theta}$ in complex plane
t	time (s)
T	tissue temperature (°C)
T_b	blood temperature (°C)
U	unit step function
w_b	blood perfusion rate (s^{-1})
W_b	blood perfusion rate multiplied by blood density, $W_b = w_b\rho_b$
x	axial coordinate (m)
z_n	constant coefficients employed in Eq. (19a)
Z	dimensionless temperature function employed in separation of variables method
Z_n	particular functions, Eq. (19a)

Greek symbols

α	tissue thermal diffusivity ($m^2\,s^{-1}$)
β	a function of Laplace domain parameter, Eq. (15)
γ	real number employed in Eq. (23)
Γ	dimensionless temperature gradient relaxation time
Γ_p	dimensionless incident heat flux exposure time
ε_n	root of β plus square of λ_n, $\sqrt{\beta + \lambda_n^2}$
ζ	dimensionless radial coordinate
ζ_{R_t}	dimensionless tissue radius
ζ_{R_p}	dimensionless incident heat flux spot radius
η	dimensionless time
θ	dimensionless tissue temperature
$\bar{\theta}$	Laplace transform of θ