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Numerical solution of an inverse reaction–diffusion problem via collocation method based on radial basis functions

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ABSTRACT

In this paper, a numerical technique is presented for the solution of a parabolic partial differential equation with a time-dependent coefficient subject to an extra measurement. This method is a combination of collocation method and radial basis functions. The operational matrix of derivative for radial basis functions is introduced and the new computational technique is used for this purpose. The operational matrix of derivative is utilized to reduce the problem to a set of algebraic equations. Some examples are given to demonstrate the validity and applicability of the new method and a comparison is made with the existing results.

1. Introduction

The parameter identification in a parabolic differential equation from the overspecified data plays an important role in engineering and physics. This technique has been widely used to determine the unknown properties of a region by measuring data only on its boundary or a specified location in the domain. These unknown properties, such as the conductivity medium, are important to the physical process, but they usually cannot be measured directly, or the process of their measurement is very expensive [1–3]. These problems are often ill-posed in the sense of Hadamard (see [4,5]), because the low random errors in measurement can lead to major errors in identifications and a direct inversion of the data for the unknown function is not possible [6]. As a result, a number of researchers have developed various methods to overcome the ill-posed nature of the inversion problem. These methods include Born approximation [7], neural-networks [8] and Levenberg–Marquardt method [9].

In this paper, authors are trying to solve an inverse problem to a class of reaction–diffusion equation using radial basis functions (RBFs) as a truly meshless/meshfree method. A meshfree method does not require a mesh to discretize the domain of the problem under consideration and the approximate solution is constructed entirely based on a set of scattered nodes. It is considered as the main advantage of these methods over the mesh dependent techniques.

1.1. Reaction–diffusion systems

A reaction–diffusion equation comprises a reaction term and a diffusion term, i.e. the typical form is as follows:

$$u_t = \nabla.(D(u;x,t)\nabla u) + f(u,\nabla u;x,t),$$

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where $u(x,t)$ is a state variable and describes temperature/concentration of a substance at position $x \in \Omega \subset \mathbb{R}^n$ at time t (Ω is an open set). D is the diffusion coefficient and f is the reaction term.

Reaction–diffusion systems are mathematical models which explain how the concentration of one or more substances distributed in space changes under the influence of two processes: local chemical reactions in which the substances are transformed into each other and diffusion which causes the substances to spread out over a surface in space. This description implies that reaction–diffusion systems are naturally applied in chemistry. However, the system can also describe dynamical processes of non-chemical nature. Examples are found in biology, geology, physics and ecology [10–12].

1.2. A class of reaction–diffusion systems

Here, we study the reaction–diffusion equation in one space dimension with $D(u;x,t) = a(t)$ and $f(u,\nabla u;x,t) = p(t)u + g(x,t)$, as following:

$$u_t = a(t)u_{xx} + p(t)u + g(x,t); \quad 0 < x < 1, \quad 0 < t < T, \tag{1}$$

subject to the initial and boundary conditions:

$$u(x,0) = u_0(x); \quad 0 \leqslant x \leqslant 1, \tag{2}$$

$$u(0,t) = g_0(t); \quad 0 \leqslant t \leqslant T, \tag{3}$$

$$u(1,t) = g_1(t); \quad 0 \leqslant t \leqslant T, \tag{4}$$

where $T > 0$ is constant and g, p, u_0, g_0 and g_1 are known functions.

In the direct problem, diffusion coefficient $a(t)$ is known function, so that the direct problem consists in finding $u(x,t)$ in the given domain whereas the discussed inverse problem consists in simultaneously finding $u(x,t)$ and the unknown coefficient $a(t)$.

Inverse problem of finding $u(x,t)$ and the unknown coefficient $a(t)$ is under-determined and we are forced to impose an additional boundary condition, such that a unique solution pair (u,a) is obtained. An additional boundary condition which can be the additional specification at a point in the spatial domain (temperature additional specification) [13], is given in the following form:

$$u(x^*,t) = E(t); \quad 0 \leqslant t \leqslant T, \tag{5}$$

where E is known function and $x^* \in (0,1)$ is constant. Employing the condition (5), a recovery of the function $a(t)$ together with the solution $u(x,t)$ can be made possible.

Therefore in this study, we solve the inverse problem (1)–(5).

Certain types of physical problems can be modeled by (1)–(5). One application is in the determination of the unknown properties in a region by measuring only data on the boundary, and particular attention has been focused on coefficients that represent physical quantities, for example, the conductivity of a medium. The methods used depend strongly on the type of equations and variables on which the unknown coefficient is assumed a priori to depend. An important but difficult case is when the unknown conductivity depends on the dependent variable of the solution u. For a heat flow problem, this has the physical interpretation of a temperature dependent on conductivity. If, however, the spatial change in the function $u(x,t)$ is small in comparison with the change in time, then a reasonable approximation to this state of affairs may be to consider the coefficient to be a function only of the time variable [14].

1.3. A brief review of other methods existing in the literature

The existence and uniqueness of the solution of this problem and more applications are discussed in [15–20]. However, the theory of the numerical solution of this problem is far from satisfactory. Cannon [19] and Jones [21] reduced the problem to a non-linear integral equation for the coefficient $a(t)$. This approach works well for parabolic equation in one space variable but does not easily extend to higher-dimensional problems because it depends on the explicit form of the fundamental solution of the heat operator. In [14], a backward Euler finite difference scheme was discussed. It is shown that this scheme is stable in the maximum norm and error estimates for u and a, and some experimental numerical results are given. Authors of [22] proved the solution of the problem for a connected domain in $\mathbb{R}^n$. In [23], this problem was studied from a different point of view. The authors first transformed a large class of parabolic inverse problems into a non-classical parabolic equation whose coefficients consist of trace type functional on the solution and its derivatives subject to some initial and boundary conditions. For the resulted non-classical problem, they introduced a variational form by defining a new function and then both continuous and discrete Galerkin procedures are employed to the non-classical problem. Author of [3] used the several explicit and implicit finite difference methods to solve this problem. In [24], an efficient pseudospectral Legendre method is developed to solve problem (1)–(5).

The authors of [25] applied the Adomian decomposition method to find solution of this problem. In [26], the numerical solution is also considered by use of Chebyshev cardinal functions; the method consists of expanding the required