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Modelling, analysis and simulation of an optical squeezer

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ABSTRACT

Squeezed states of light have numerous potential applications ranging from gravitational wave detection, quantum teleportation, quantum cryptography and quantum communication. They are generated using an optical squeezer and possess the desirable property of having less noise in one quadrature than dictated by the quantum noise limit (QNL). In this paper, we model the nonlinear optical process by which squeezed light is generated in a classical framework and analyze its steady-state behavior from a control theoretic perspective. In particular, through computer simulation, we provide a visual perspective of the effect of the steady-state operating point of the optical squeezer on the type of squeezed states of light generated.

1. Introduction

It is well known in the physics literature that experiments performed at the quantum level are restricted by the quantum noise limit (QNL) [1]. The QNL sets a fundamental limit to the performance of quantum systems and puts a bound on the signal to noise ratio that can be achieved, restricting the usefulness of applications involving quantum technology. Advances in nano-technology and atomic systems have led to devices which are sensitive enough to detect quantities at a level where quantum effects become significant. Furthermore, with the use of nonlinear effects and nonlinear media in quantum optics [2,3], it is possible to change the statistics of quantum noise, allowing for measurements that are better than permitted by the standard quantum limit (SQL). Here, SQL refers to the minimum level of quantum noise that can be obtained with the use of states of light that possess standard quantum noise properties.

If we define the amplitude and phase quadrature operators [4] of an optical field as $\mathbf{X}^{+}$ and $\mathbf{X}^{-}$ respectively, then the Heisenberg uncertainty principle [1,5] implies that the variance of the operators in any given state is constrained by

$$\langle \Delta \mathbf{X}^{+^2} \rangle \langle \Delta \mathbf{X}^{-^2} \rangle \geqslant 1; \tag{1}$$

where $\langle \cdot \rangle$ denotes the quantum expectation value. In the case of laser light, which can be modeled as being in a coherent state $|\alpha\rangle$ [4] or minimum uncertainty state (with a minimal product of the amplitude and phase quadrature uncertainties), the uncertainty associated with the quantum state is equally divided between the two quadratures such that

$$\langle \Delta \mathbf{X}^{+^2} \rangle = \langle \Delta \mathbf{X}^{-^2} \rangle = 1. \tag{2}$$

The uncertainty can thus be visualized as shown in Fig. 1, in what is known as the *ball on stick* diagram. The straight line is the coherent amplitude and the diameter of the ball (which has been blown up in scale in the picture) represents the

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Nomenclature

QNL	quantum noise limit
SQL	standard quantum limit
GWD	gravitational wave detectors
EPR	Einstein–Podolsky–Rosen
OPA	optical parametric amplifier
OPO	optical parametric oscillator
SHG	second harmonic generation
DFG	difference frequency generation

variance of the respective quadratures. It is circular in this case indicating that noise in each quadrature is equal. The "uncertainty ball" arises as a result of the quantum nature of light, affecting a given system and restricts how close two measurable quantum states can be if they are to be distinguished from each other. Indeed, classical noises affect measurement of classical states in a similar fashion but they can however (in theory) be completely suppressed with improved measurement techniques. This is not the case for quantum noises whose sizes are dictated by the laws of quantum mechanics rather than as a result of the limitations of the measurement devices. It is possible, fortunately to partly circumvent this problem by concentrating the quantum noises in specific quadratures of light. In other words, we can rearrange the noise distribution between the quadratures such that one quadrature has less noise than the other. Quantum states of light which have this property of asymmetric quadrature noise distributions are known as "squeezed" or "non-classical" states of light.

2. Squeezed states

Some of the different types of squeezed states that can be achieved from a coherent state are shown in Fig. 2. As mentioned before, squeezed states of light have their quantum noises squeezed and possess noise distribution which are less than those of the coherent states of light in *specified directions*.

A coherent state of light has a Gaussian distribution such that (2) is satisfied. Pauli [6] showed that both a coherent quantum state and squeezed state can be completely described using only 2-dimensional Gaussian distribution functions. For a squeezed state, the only requirement is that the width of these functions need to satisfy the uncertainty principle for any chosen direction θ. The contour line for such a 2-dimensional function turns out to be always elliptical, while that for a coherent state is circular. The total area inside the contour can however never be less than that of the uncertainty area of a coherent state. For the case of phase squeezed states (Fig. 2(b)), they have a narrow axis aligned with the phase axis and the widest distribution orthogonal to the phase axis. Similarly, for an amplitude squeezed state (Fig. 2(c)), there is less noise along the amplitude quadrature and the fluctuations along the phase quadrature are above the standard quantum noise limit. In the presence of external sources of noise, both quadratures will be affected but as long as the excess noise introduced does not drive the fluctuations above the standard quantum noise limit, squeezed states are obtained. In this way, depending on whether the amplitude or the phase quadrature is being measured, the state can be squeezed accordingly such that the measurement can become more sensitive in a given direction. Moreover, a quantum state can be squeezed along any arbitrary angle with respect to the predefined amplitude and phase quadratures, as shown in Fig. 2(d) and the state is then described as quadrature squeezed state. Squeezed states are of course, going to be most useful in applications where the standard quantum noise limit has already been reached.

2.1. Applications of squeezed states

Squeezed states of light have a wide range of potential applications. One promising application of squeezed states of light is in gravitational wave detection [7,8], where it can potentially improve the sensitivity of interferometers. Gravitational

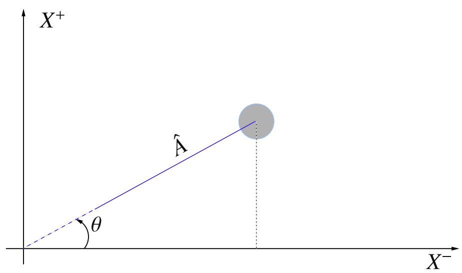

Fig. 1. Minimum uncertainty state.