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Application of smart collocation method for solving strongly nonlinear boundary value ordinary differential equations

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ABSTRACT

In this study, a new simple technique called, "smart collocation" is presented. This method is an extension of the collocation method for solving strongly nonlinear boundary value ordinary differential equations. Effects of varying the iteration steps, degree of nonlinearity and position of the collocation points in each sub-interval are investigated. To verify, the calculated results are compared with the bvp4c MATLAB function results. It is shown that, in the case of strongly nonlinear equations, more accurate results can be achieved by moving the position of the collocation points in sub-intervals. For a general problem, based on the degree of nonlinearity, a criterion for selecting appropriate collocation points is obtained. It is shown that varying the position of collocation points or simply smart collocation method can obviously increase accuracy of the solution.

1. Introduction

Every physical process is a nonlinear system and should be modeled using nonlinear equations. Many practical examples of nonlinear dynamic behavior have been reported in the engineering literature. Brake squeal in the automotive industry, backlash and friction in surfaces and joints, saturation effects in hydraulic actuators, bearings and guide ways friction in mechatronic systems, contact problem in vibro-impact systems and large deformed structures are some of the nonlinear cases in mechanical engineering problems [1,2].

The boundary value problem (BVP) in nonlinear ordinary differential equations (ODEs) has many applications in mechanical sciences [3–5]. Methods of solving the BVP nonlinear ODEs have been studied for several years [6,7]. Siraj ul et al. used a local radial basis functions collocation method to investigate the transient nonlinear coupled Burgers' equations [8]. Frind and Pinder provided a powerful collocation method for solving potential problems in irregular domains [9]. Aziz et al. proposed two efficient numerical methods to solve elliptic partial differential equations [10]. They showed that their methods can accurately be applied to study the Neumann's boundary conditions. Tohidi et al. presented a direct solution method to solve the generalized pantograph equation [11]. In doing so, they used the Bernoulli operational matrix and showed that even small dimension of Bernoulli operational matrix can lead to satisfactory results. They showed that accuracy of results, which are obtained using the collocation method, can be better than the Galerkin solution. Chi et al. presented a strong weighted form of collocation method. They mixed radial basis approximations to investigate the pressure and displacement fields of incompressible linear elasticity [12]. Hu et al. presented weighted radial basis collocation method for boundary value problems [13]. They demonstrated that the weighted radial basis collocation method can significantly improve

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accuracy and convergence rates of the numerical solution. Messner and Schanz offered a collocation boundary element method for linear poroelasticity. Their model is presented based on the first boundary integral equation with only weakly singular kernels [14]. Aluru provided a point collocation method based on reproducing kernel approximations [15]. He showed that one and two-dimensional problems can accurately be solved using the point collocation method.

Nowadays, there is a large tendency toward numerical simulation of nonlinear systems. The reason for this interest lies in the growth of powerful computers. In the present study, a simply programmed method is presented to solve an important practical nonlinear BVP. Collocation method is preferred to investigate the discussed problem, because:

- The collocation methods can decrease computational efforts [3].
- Convergence properties of this class of methods are completely helpful [16].
- Unlike the collocation method, other methods (i.e. shooting methods) have several limitations for applying to unlimited classes of singular problems [17].

It is shown that an equidistant spacing between collocation points is not generally appropriate [18]. Furthermore, in the present study, it is shown that ordinary collocation methods cannot accurately solve strongly nonlinear problems. In this study, the so-called "smart collocation" technique is proposed to eliminate the discussed shortcoming of the ordinary collocation methods. Interior collocation, boundary collocation and mixed collocation are three classes of the collocation method, which are used in previous researches. In these collocation methods, location of the collocation points in each sub-interval is fixed.

Studying the effect of changing the location of the collocation points on the accuracy of solution is the main objective of the present study. In doing so, in the smart collocation method, during the solution procedure, position of the collocation points in each sub-interval, varies with the problem degree of nonlinearity. It should be noted that, the smart collocation deals, mainly with interior and boundary collocation methods. Finally, the smart collocation method is demonstrated using a practical example and the calculated results are verified by the result of the bvp4c MATLAB function.

2. Problem statement

2.1. Objective and overview of the present study

The goals of this research are threefold: (1) to investigate the feasibility of applying the collocation method to solve a practical nonlinear ODE, (2) to investigate effects of location of the collocation points in each sub-intervals on accuracy of the solution, and (3) to present a new collocation-based method to increase accuracy of the solution.

2.2. Nonlinear differential equation

Consider the following non-homogenous nonlinear differential equation:

$$L\theta = \theta_{,ss} - \kappa^2 \cos 2\theta / 2\sqrt{2} = \kappa^2/\sqrt{2}; \quad \theta(0) = 0,\ \theta(1) = 0 \tag{1}$$

where κ is a coefficient and in this study, it shows the degree of nonlinearity. This equation can be easily changed to the following important nonlinear equation, which describes several vibration and stability problems of the structures [19–21]:

$$\theta_{,ss} + \kappa^2 \sin\theta = 0; \quad \theta(0) = 0,\ \theta(1) = 0 \tag{2}$$

The above equation can be used to represent the structural and vibration behavior of a largely deformed column and largely excited pendulum, respectively. Using the trigonometric identities and the binomial theorem, it can be concluded that $\sin\theta = (1 - 0.5\cos(2\theta) + \ldots)/\sqrt{2}$. Substituting this equation into Eq. (2) shows that Eqs. (1) and (2) are relatively the same.

3. Solution method

In this section, the ordinary collocation procedure is used to solve the nonlinear BVP, which is presented in Eq. (2). In doing so, piecewise polynomial functions are used to describe the solution in each sub-interval. For adapting the polynomial functions to the exact solution, two collocation points are used in each sub-interval. Unlike the smart collocation method, in the ordinary collocation method, the location of the collocation points in each sub-interval is fixed.

In this section, the piecewise polynomial functions, which are used in the present study, are introduced. Then the ordinary collocation procedure is discussed. Finally, method of selecting location of the collocation points is presented.

3.1. Piecewise polynomial functions

Consider the following interval partition:

$$a = x_1 < x_2 < \cdots < x_{l+1} = b. \tag{3}$$