Applied Mathematical Modelling 37 (2013) 7983–8003

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Stress and strain-inertia gradient elasticity in free vibration analysis of single walled carbon nanotubes with first order shear deformation shell theory

F. Daneshmand [a,b,c,*], M. Rafiei [d], S.R. Mohebpour [d], M. Heshmati [e]

[a] Department of Mechanical Engineering, McGill University, 817 Sherbrooke Street West, Montreal, Quebec, Canada H3A 2K6
[b] Department of Bioresource Engineering, McGill University, 21111 Lakeshore Road, Sainte-Anne-de-Bellevue, Quebec, Canada H9X 3V9
[c] School of Mechanical Engineering, Shiraz University, Shiraz, Iran
[d] Department of Mechanical Engineering, School of Engineering, Persian Gulf University, Bushehr 75168, Iran
[e] Department of Mechanical Engineering, Kermanshah University of Technology, Kermanshah, Iran

ARTICLE INFO

Article history:
Received 30 May 2012
Received in revised form 7 December 2012
Accepted 10 January 2013
Available online 20 March 2013

Keywords:
Gradient elasticity
Carbon nanotubes
Shell theory
Vibration analysis

ABSTRACT

A gradient-enriched shell formulation is introduced in the present study based on the first order shear deformation shell model and the stress gradient and strain-inertia gradient elasticity theories are used for dynamic analysis of single walled carbon nanotubes. It provides extensions of the first order shear deformation shell formulation with additional higher-order spatial derivatives of strains and stresses. The higher-order terms are introduced in the formulation by using the Laplacian of the corresponding lower-order terms. The proposed shell formulation includes two length scale size parameters related to the strain gradients and inertia gradients. The effects of the transverse shear, aspect ratio, circumferential and half-axial wave numbers and length scale parameters on different vibration modes of the single-walled carbon nanotubes are elucidated. The results are also compared with those obtained from a classical shell theory with Sanders–Koiter strain-displacement relationships.

1. Introduction

Carbon nanotubes (CNTs) are one of the most promising products of nanotechnology with Young's modulus about five times higher than that of steel and various applications in disease diagnosis and environmental monitoring [1]. Investigation of vibration characteristics of CNTs is of practical importance [2–5]. The majority of previous studies on vibration analysis of carbon nanotubes (CNTs) can be classified into two main categories. In the first category, theory of continuum mechanics is used to develop the equations of motion of the problem whereas the small scale effects due to atomic scale of the lattice structures of CNTs are not taken into account. In the classical continuum mechanics, the stress at a given point is only related to the strain at the same point. However, the heterogeneity effects in nanostructures are not reasonably be accounted for by direct application of classical continuum mechanics. Moreover, the length scales associated with nanotubes are often sufficiently small to call the applicability of classical continuum models into question. Higher order elasticity theories containing internal material length scale parameters attempt to extend the continuum approach to smaller length scales. In the second category, the analysis of CNTs is accomplished by using modified elasticity formulations including nonlocal and strain-inertia gradient effects to predict the static and dynamic behavior of CNTs.

* Corresponding author at: Department of Mechanical Engineering, McGill University, 817 Sherbrooke Street West, Montreal, Quebec, Canada H3A 2K6. Tel.: +1 514 630 7994.

E-mail addresses: farhangdaneshmand@gmail.com, farhang.daneshmand@mcgill.ca (F. Daneshmand).

0307-904X/$ - see front matter
http://dx.doi.org/10.1016/j.apm.2013.01.052

As a higher order continuum theory, the nonlocal elasticity theory has been developed by Eringen [6] and has been widely used to study the size effect on the vibration properties of CNTs. This theory assumes that the stress at a given point is related to the strain at all points in the body. Peddison et al. [7] used a nonlocal version of Euler–Bernoulli beam theory to estimate the importance of nonlocal effects by solving four representative problems. Murmu and Padhan [8] utilized nonlocal elasticity theory and differential quadrature method to analyze the vibration response of a nano-cantilever beam with a non-uniform cross-section. The transverse free vibration of an axially loaded non-uniform single-walled carbon nanotube based on a nonlocal Rayleigh beam has been investigated by Mustapha and Zhong [9]. Demir et al. [10] considered free vibration of CNTs by using the shear deformation beam theory and discrete singular convolution method. In another work, they have studied the effects of small scale parameter on bending and vibration of microtubules (MTs) [11]. A new approach for obtaining accurate bending moments and displacements in microtubules (MTs) in a computationally efficient manner by using nonlocal continuum theory has also been presented [12]. Setoodeh and Khosrownejad [13] used the Euler–Bernoulli beam theory and the nonlocal elasticity model based on Eringen's formulation and studied the post-buckling configurations of single-walled carbon nanotubes with different boundary conditions. Ansari et al. [14] considered classical beam theories with different boundary conditions and studied small scale effects on free vibration characteristics of SWCNTs using nonlocal elasticity formulation. More recently, Rafiei et al. [15] used the nonlocal Euler–Bernoulli beam theory to study the influence of the internal moving fluid on vibration characteristics of non-uniform single-walled carbon nanotubes conveying fluid and embedded in a viscoelastic medium. The effects of taper ratio of single-walled carbon nanotube, small-scale parameter and viscoelastic medium on critical steady flow velocity have also been investigated. It is worth noting that the strategy used in the above mentioned works is to apply the classical equilibrium equations and replace relevant quantities with the corresponding nonlocal quantities. These works may be classified as the "partial nonlocal stress models" as proposed by Lim et al. [16]. As another approach and by using the Laplacian of stress or strain, the standard constitutive equations of elasticity can be generalized in the gradient elasticity theory. In the gradient elasticity theories, extensions to the classical equations of elasticity are provided by using the additional higher-order spatial derivatives of strains, stresses and/or accelerations. A number of gradient elasticity formulations have been presented in literature to model the dispersion of flexural waves in carbon nanotubes with Euler–Bernoulli and Timoshenko assumptions [17–19]. Eringen's nonlocal elasticity theory [6] can be considered as a stress gradient elasticity theory obtained from the gradient elasticity theory. It should be noted that the gradient elasticity models with strain and inertia gradients can be used to predict the dispersive characteristics of CNTs. More recently, Wang [20] compared two gradient elasticity formulations with stress and combined strain-inertia gradients for dynamic analysis of nanotubes convoying fluid. Moreover, Ansari et al. [21] investigated the vibration behavior of single-walled carbon nanotubes (SWCNTs) by using the different gradient beam theories. They implemented the Euler–Bernoulli and Timoshenko beam theories in conjunction with the gradient elasticity theories including stress, strain and combined strain-inertia to study the free vibration of SWCNTs.

The stress gradient beam models has one material length scale parameter whereas the combined strain-inertia gradient beam models include two material length scale parameters related to the inertia and strain gradients and can also be used to investigate the size effect on the dynamical behavior of nanotubes.

The motivation of the present study comes from the fact that the nonlocal elasticity theory is essentially a stress gradient elasticity theory [17] and it is quite sufficient for static problems whereas it might be inadequate in dynamic analysis of nanotubes.

According to our knowledge, no investigation has been performed on vibration of SWCNTs with first order shear deformation shell theory (FSDT) based on the stress and strain-inertia gradient elasticity theories. In the present paper, the stress gradient (SG) and strain-inertia gradient (SIG) elasticity theories are implemented to investigate the effects of material length scales on the frequency analysis of a SWCNT. The effects of the transverse shear, aspect ratio, circumferential and half-axial wave numbers and length scale parameters on the eigenfrequencies of the single-walled carbon nanotubes are also elucidated. The applicability of the method for vibration analysis of SWCNTs is investigated by comparing the results with those obtained from classical shell theory for two types of single-walled carbon nanotubes (20, 20) and (15, 15). The validity of the proposed method is verified by comparing the numerical results with available known results for vibration of microtubule in a special case.

2. Gradient elasticity shell models for single-walled carbon nanotubes

In this paper, the first-order shear deformation shell theory based on the stress gradient (SG) and strain-inertia gradient (SIG) elasticity theories for isotropic materials are used to investigate the dynamic behavior of the single-walled carbon nanotubes (SWCNTs). To study the effects of the transverse shear on vibration frequencies of SWCNTs, the results of the first order shear deformation shell theory are compared with those predicted by the classical shell theory (CST) with Sanders–Koiter strain-displacement relationships by neglecting the nonlinear terms.

2.1. Stress gradient elasticity theory

The stress gradient elasticity or the nonlocal elasticity has been represented by Eringen [22]. In contrast to the classical elasticity theory, the stress at a given point in the nonlocal elasticity theory is related to the strain at all points in the body and the linear constitutive equation of the nonlocal, isotropic, elastic solid is expressed in an integral form as [6,22]