Applied Mathematical Modelling 37 (2013) 8131–8140

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

An SIR epidemic model with time-varying pulse control schemes and saturated infectious force

Yuying He [a], Shujing Gao [a,b,*], Dehui Xie [a]

[a] Key Laboratory of Jiangxi Province for Numerical Simulation and Emulation Techniques, College of Mathematics and Computer Science, Gannan Normal University, Ganzhou 341000, PR China
[b] National Institute of Parasitic Diseases, Chinese Center for Disease Control and Prevention, Shanghai 200025, PR China

ARTICLE INFO

Article history:
Received 20 January 2012
Received in revised form 13 December 2012
Accepted 17 March 2013
Available online 26 March 2013

Keywords:
Time-varying pulse vaccination
Time-varying pulse treatment
Saturation incidence
Threshold value

ABSTRACT

Vaccination and treatment policies are two methods used to control the spread of disease. In this paper, we study an SIR epidemic model with time-varying pulse vaccination of the susceptible and time-varying pulse treatment of the infected population. Owing to the seasonal fluctuations in transmission of many disease, we propose the model with periodic saturation incidence. The Threshold value R_0 is established. We demonstrate that the disease dies out for $R_0 < 1$, whereas it is permanent for $R_0 > 1$.

1. Introduction

It is well known that epidemic diseases have tremendous influence on human life. Every year, millions of people around the world suffer from or die of various epidemic diseases. And a great deal of human and economical resources are devoted to epidemic diseases research for a long time. So controlling epidemic diseases has been an increasingly complex issue. The standard conventional approach has been constant vaccination, or a uniform and continuous effort of administering the vaccine to a population, which is an effective way to control the transmission of diseases and was considered in many literatures [1–7]. However, considering the cost and the side effect of the medications, especially for infants, many medications have potential harmful effects on them, we prefer to the impulsive vaccination strategy, which is easier to be manipulated and has the relatively low cost. The control strategy of pulse vaccination is based on the strategy of applying vaccinations periodically to a large fraction of the population, who will become removed with permanent or temporary acquired immunity in a very short time. The theoretical study of pulse vaccination strategy was firstly introduced by Agur and coworkers in [5]. From then on, series of compartment models with pulse vaccination have been proposed and analyzed (For example [8–12]).

In addition, the treatment is an important method to decrease the spread of diseases. In classical epidemic models, the treatment rate of the infective is assumed to be proportional to the number of the infective population, that is, it is usually assumed that a fraction $0 \leqslant p \leqslant 1$ of infected individuals are successfully treated continuously in time. Recently Liu and Stechlinski [13] proposed an SIR epidemic model with pulse treatment of the infected and pulse vaccination of the susceptible. They assume that impulses are successfully applied periodically to a portion $0 \leqslant p \leqslant 1$ of the susceptible population through vaccinations every T time units, and then they translate into the recovered class. Assume that the same portion of the infected population is successfully treated impulsively every T time units, who then translate into the recovered class.

* Corresponding author.
E-mail address: gaosjmath@126.com (S. Gao).

0307-904X/$ - see front matter © 2013 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.apm.2013.03.035

Motivated by seasonal changes in the transmission of an infectious disease, they also assume that the variations in the contact rate are periodic. They considered an SIR model as following:

$$\begin{cases} \left.\begin{aligned} \frac{dS(t)}{dt} &= \mu - \beta_i SI - \mu S, \\ \frac{dI(t)}{dt} &= \beta_i SI - gI - \mu I, \\ \frac{dR(t)}{dt} &= gI - \mu R, \end{aligned}\right\} & t \in (t_{k-1}, t_k), \\ \left.\begin{aligned} S(t^+) &= (1-p)S(t), \\ I(t^+) &= (1-p)I(t), \\ R(t^+) &= R(t) + pS(t) + pI(t), \end{aligned}\right\} & t = kT, \end{cases} \tag{1.1}$$

where $k = 1, 2, 3, \ldots,\ i = 1, 2, \ldots, m$, $T = \tau_1 + \tau_2 + \cdots + \tau_m$ and $\tau_k = t_k - t_{k-1}$. And they discussed the condition that the solution of system (1.1) converges to the periodic disease-free solution.

The incidence rate plays an important role in modeling epidemic diseases. In many epidemic models, the bilinear incidence rate βSI and standard incidence rate $\beta SI/N$ are frequently used. In order to model the contagion process better, many investigations have been done to extend this law which focuses on modeling non-linear dependencies on Susceptibles, such as the saturated infectious law $\beta SI/(1+\alpha S)$, it tends to $\beta I/\alpha$ as $S \to \infty$. In the past three decades, epidemiology of infectious diseases has focused on the theoretical and the numerical-experimental study of the effects of periodically varying external factors on the time course of the incidence of infectious diseases [14]. It is found that there are some different mechanisms in driving seasonal transmission of human infectious diseases, including pathogen appearance and disappearance, environmental changes, and host-behavior changes, and so on [15–17]. Thus, it is reasonable to consider the periodic (seasonal) fluctuations in transmission. The study of epidemic models with periodic and variant contact rate has recently been carried out enthusiastically for example [18,19] and reference cited therein. Gao et al. [20] also studied an SIR model with periodic and variant saturation incidence $\beta(t)SI/(1+\alpha S)$.

Owing to the influence of seasonality on incidence of infectious diseases and in order to utilize limited resources more effectively, generally speaking, we need to consider the pulse control time intervals are irregular in a specific period. Thus, we naturally consider some diseases models with time-varying pulse control schemes. Note that pulse vaccination rate and pulse treatment rate may be varied at every pulsing time in one period in our real life. Motivated by these works, we formulate a model for including periodic saturation incidence, time-varying pulse vaccination of the susceptible and time-varying pulse treatment of the infected as the following:

$$\begin{cases} \left.\begin{aligned} \frac{dS(t)}{dt} &= \mu - \frac{\beta(t)SI}{1+\alpha S} - \mu S, \\ \frac{dI(t)}{dt} &= \frac{\beta(t)SI}{1+\alpha S} - gI - \mu I, \\ \frac{dR(t)}{dt} &= gI - \mu R, \end{aligned}\right\} & t \neq t_i, \\ \left.\begin{aligned} S(t^+) &= (1-\theta_i)S(t), \\ I(t^+) &= (1-p_i)I(t), \\ R(t^+) &= R(t) + \theta_i S(t) + p_i I(t), \end{aligned}\right\} & t = t_i, \end{cases} \tag{1.2}$$

where $0 \leqslant p_i \leqslant 1$ are the time-varying pulse vaccination rates, $0 \leqslant \theta_i \leqslant 1$ are the time-varying pulse treatment of the infected rates, and t_i is the time of pulsing and satisfying $0 = t_0 < t_1 < t_2 < \cdots$, and $\lim_{i\to\infty} t_i = \infty$. It needs to be pointed out that the variables $S(t), I(t), R(t)$ represent the proportion of susceptible, infected and recovered individuals at time t in the human population, respectively, so that $R(t) + S(t) + I(t) \equiv 1$. The meaningful physical domain for this system is $\Gamma = \{(S, I, R) \in R_+^3 | S + I + R = 1\}$. The initial conditions are $S(0^+) = S_0 > 0,\ I(0^+) = I_0 > 0,\ R(0^+) = R_0$ such that $(S_0, I_0, R_0) \in \Gamma$. We also assume the following assumptions hold.

(H1) $t_i - t_{i-1} = \tau_i,\ t_m = \sum_{i=1}^m \tau_i = T,\ t_{i+m} = t_i + T,\ p_{i+m} = p_i,\ \theta_{i+m} = \theta_i,\ \beta(t+T) = \beta(t)$, here m is a fixed integer.

Noting that $R(t) = 1 - S(t) - I(t)$, the system (1.2) can be determined by the following system

$$\begin{cases} \left.\begin{aligned} \frac{dS(t)}{dt} &= \mu - \frac{\beta(t)SI}{1+\alpha S} - \mu S, \\ \frac{dI(t)}{dt} &= \frac{\beta(t)SI}{1+\alpha S} - gI - \mu I, \end{aligned}\right\} & t \neq t_i, \\ \left.\begin{aligned} S(t^+) &= (1-\theta_i)S(t), \\ I(t^+) &= (1-p_i)I(t), \end{aligned}\right\} & t = t_i \end{cases} \tag{1.3}$$

with initial conditions $S(0^+) = S_0 > 0,\ I(0^+) = I_0 > 0$.

In this paper, our main purpose is to establish a threshold of system (1.3), which determines permanence and extinction of the disease. And the paper is organized as follows: in the next section, we state some lemmas which will be essential to our proofs. In Section 3, we give the threshold condition and analyze the extinction of the disease. The permanence of the disease is discussed in Section 4. We end the paper with the numerical simulations and a brief discussion in Section 5.