Applied Mathematical Modelling 38 (2014) 1597–1606

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

A new approach of the Chebyshev wavelets method for partial differential equations with boundary conditions of the telegraph type

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ARTICLE INFO

Article history:
Received 8 December 2012
Accepted 10 September 2013
Available online 27 September 2013

Keywords:
Partial differential equations
Chebyshev wavelets
Operational matrix of integration
Operational matrix of differentiation
Telegraph equation

ABSTRACT

In this paper, we develop an accurate and efficient Chebyshev wavelets method for solution of partial differential equations with boundary conditions of the telegraph type. In the proposed method we have employed mutually the operational matrices of integration and differentiation to get numerical solutions of such equations. The power of this manageable method is confirmed. Moreover the use of Chebyshev wavelet is found to be accurate, simple and fast.

1. Introduction

Wavelet methods have been applied for solving partial differential equations (PDEs) from the beginning of 1990s [1]. In the last two decades this method of solution for such problems has attracted great attention and numerous papers about this topic have been published. Due to this fact we must confine somewhat our analysis; in the following only PDEs of mathematical physics and of electrostatics are considered. From the first field of investigation [2–7] can be cited. For elasticity problems we refer to [8–14]. In these papers different wavelet families have been applied. In most cases the wavelet coefficients have been calculated by the Galerkin or collocation method, for which we have to evaluate integrals of some combinations of the wavelet functions. We consider the second order linear hyperbolic telegraph equation in one-dimension of the following form:

$$\frac{\partial^2 u}{\partial t^2} + 2\alpha\frac{\partial u}{\partial t} + \beta^2 u = \frac{\partial^2 u}{\partial x^2} + f(x,t), \quad a \leqslant x \leqslant b, \quad t \geqslant 0, \quad \alpha > \beta > 0, \tag{1}$$

where α and β are known constants. This equation is commonly used in the study of wave propagation of electric signals in a cable transmission line and also in wave phenomena. This equation has been also used in modeling the reaction–diffusion processes in various branches of engineering sciences and biological sciences by many researchers (see [15] and references therein). Moreover this equation represents a damped wave motion for $\alpha > 0$ and $\beta = 0$. In recent years, much attention has been given in the literature to the development analysis and implementation of stable methods for the numerical solution of

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http://dx.doi.org/10.1016/j.apm.2013.09.013

second-order hyperbolic equations, especially, telegraph equation, that is very important in engineering sciences (see for example [15] and references therein). The aim of the present work is to develop Chebyshev wavelets method with the operational matrices of integration and differentiation, mutually for solving partial differential equations with boundary conditions of the telegraph type, which is fast and mathematically simple and guarantees the necessary accuracy for a relative small number of grid points. The outline of this article is as follows: In Section 2 we describe properties of Chebyshev wavelets. In Section 3 the proposed method is used to approximate solution of the problem. In Section 4 some numerical examples are solved by applying the method of this article. Finally a conclusion is drawn in Section 5.

2. Chebyshev wavelets and their properties

Chebyshev wavelets $\psi_{n,m}(t) = \psi(k,\hat{n},m,t)$ have four arguments; $k \in \mathbb{N}, n = 1,2,\ldots,2^{k-1}$, and $\hat{n} = 2n-1$, moreover m is the degree of the Chebyshev polynomial of the first kind and t is the normalized time i.e. $t \in [0,1)$. They are defined on the interval $[0,1)$ as:

$$\psi_{n,m}(t) = \begin{cases} 2^{k/2}\tilde{T}_m(2^k t - \hat{n}), & \frac{\hat{n}-1}{2^k} \leqslant t < \frac{\hat{n}+1}{2^k}, \\ 0, & \text{otherwise}, \end{cases} \tag{2}$$

where

$$\tilde{T}_m(t) = \begin{cases} \frac{1}{\sqrt{\pi}}, & m = 0, \\ \sqrt{\frac{2}{\pi}}T_m(t), & m > 0, \end{cases} \tag{3}$$

$m = 0,1,\ldots,M-1$, and M is a fixed positive integer. Here, $\{T_m(t), m \in \mathbb{N} \cup \{0\}\}$ is the set of well-known Chebyshev polynomials of degree m which are orthogonal with respect to the weight function $w(t) = 1/\sqrt{1-t^2}$ on the interval $[-1,1]$. We should note that in dealing with Chebyshev polynomials the weight function $\tilde{w}(t) = w(2t-1)$ have to be dilated and translated as $w_n(t) = w(2^k t - \hat{n})$ to get orthogonal wavelets.

Any square integral function $f(t)$ defined over $[0,1)$ may be expanded by Chebyshev wavelets as:

$$f(t) = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} c_{nm}\psi_{nm}(t), \tag{4}$$

where $c_{nm} = (f(t), \psi_{nm}(t))$ and $(,)$ denotes the inner product.

If the infinite series in (4) is truncated, then (4) can be written as:

$$f(t) \simeq \sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c_{nm}\psi_{nm}(t) = C^T\Psi(t), \tag{5}$$

where C and $\Psi(t)$ are $\hat{m} = (2^{k-1}M)$ column vectors.

For simplicity, we write (5) as:

$$f(t) \simeq \sum_{i=1}^{\hat{m}} c_i\psi_i(t) = C^T\Psi(t), \tag{6}$$

where $c_i = c_{nm},\ \psi_i(t) = \psi_{nm}(t)$.

The index i, is determined by the relation $i = M(n-1)+m+1$. Therefor we have:

$$\begin{aligned} &C \triangleq [c_1, c_2, \ldots, c_{\hat{m}}]^T, \\ &\Psi(t) \triangleq [\psi_1(t), \psi_2(t), \ldots, \psi_{\hat{m}}(t)]^T. \end{aligned} \tag{7}$$

Similarly, an arbitrary function of two variables $u(x,t)$ defined over $[0,1) \times [0,1)$, may be expanded into Chebyshev wavelets basis as:

$$u(x,t) \simeq \sum_{i=1}^{\hat{m}}\sum_{j=1}^{\hat{m}} u_{ij}\psi_i(x)\psi_j(t) = \Psi^T(x)U\Psi(t), \tag{8}$$

where $U = [u_{ij}]$ and $u_{ij} = \big(\psi_i(x), \big(u(x,t), \psi_j(t)\big)\big)$.

Taking the collocation points $t_i = \frac{(2i-1)}{2\hat{m}}\,(i = 1,\ldots,\hat{m})$, in (7), we define the wavelet matrix $\Phi_{\hat{m}\times\hat{m}}$ as:

$$\Phi_{\hat{m}\times\hat{m}} = \left[\Psi\left(\frac{1}{2\hat{m}}\right), \Psi\left(\frac{3}{2\hat{m}}\right), \ldots, \Psi\left(\frac{2\hat{m}-1}{2\hat{m}}\right)\right]. \tag{9}$$

Indeed $\Phi_{\hat{m}\times\hat{m}}$ has a diagonal form [16].