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Homotopy perturbation method for linear programming problems

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ABSTRACT

In this paper, He's homotopy perturbation method (HPM) is applied for solving linear programming (LP) problems. This paper shows that some recent findings about this topic cannot be applied for all cases. Furthermore, we provide the correct application of HPM for LP problems. The proposed method has a simple and graceful structure. Finally, a numerical example is displayed to illustrate the proposed method.

1. Introduction

Experiential surveys show that linear programming (LP) is one of the most important techniques in applied mathematics. Many real-world problems can be transformed to linear programming model. Hence this model is an indispensable tool in today's applications such as energy, military, transportation, manufacturing, etc. There are many methods for solving LP (see [1] and references therein). Recently, Mehrabinezhad and Saberi-Nadjafi [2], presented a new method to solve LP based on the homotopy perturbation method (HPM). HPM was first proposed by He in 1998 [3] and was further developed and improved by him (see, [3–7] and the references therein). He, presented a homotopy perturbation technique based on the introduction of homotopy in topology, coupled with the traditional perturbation method for the solution of algebraic equations. This technique provides a summation of an infinite series with easily computable terms, which converges rapidly to the solution of the problem. In the literature, various authors have successfully applied HPM for many kinds of different problems such as nonlinear partial differential equations [8,9], nonlinear integral and integro-differential equations [10–12], fractional IVPs [13], nonlinear systems [14] and also linear systems [15,16]. For details of the number of publications on HPM according to web of science, we refer to [7, Fig. 1].

In this paper we study the homotopy perturbation method and point out some problems in the article [2]. Furthermore, we use HPM for solving linear programming (LP) problems under unrestricted variables. This paper is organized as follows. In Section 2, we present some notations and a brief review of the homotopy perturbation model for solving linear system. In Section 3, we point out the problems of the mentioned paper. In Section 4, we introduce application of HPM for LP under unrestricted variables with some examples.

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2. Homotopy perturbation method (HPM) for linear systems

Consider the following linear equation:

$$Ax = b, \tag{2.1}$$

where,

$$A = [a_{ij}], \quad b = [b_i] \quad x = [x_j], \quad i = 1, 2, \ldots, n \quad j = 1, 2, \ldots, n.$$

Let $L(u) = Au - b$ and $F(u) = u - w_0$, where w_0 is a known vector.

Then we define *homotopy* $H\,(u,p)$, as follows:

$$H(u,p) = (1-p)F(u) + \alpha p L(u) = 0, \tag{2.2}$$

where $p \in [0,1]$ is an embedding parameter and α is non-zero auxiliary parameter.

Obviously, we will have;

$$H(u,0) = F(u), \tag{2.3}$$
$$H(u,1) = L(u). \tag{2.4}$$

According to the HPM, we can first use the embedding parameter p as a small parameter, and assume that the solution of Eq. (2.1) can be written as a power series in p:

$$u = u_0 + pu_1 + p^2u_2 + \cdots \tag{2.5}$$

and the exact solution is obtained as follows:

$$x = \lim_{p\to 1} u = \lim_{p\to 1}(u_0 + pu_1 + p^2\,u_2 + p^3\,u_3 + \cdots) = \sum_{j=0}^{\infty} u_j. \tag{2.6}$$

Putting Eq. (2.5) into Eq. (2.2), and comparing the coefficients of identical degrees of p on both side, we find,

$$\begin{aligned}
&p^0 : u_0 = w_0 \\
&p^1 : (\alpha A - I)u_0 + u_1 - w_0 - \alpha b = 0, \quad u_1 = \alpha b - (\alpha A - I)u_0 + w_0, \\
&p^2 : (\alpha A - I)u_1 + u_2 = 0, \quad u_2 = -(\alpha A - I)u_1, \\
&\vdots
\end{aligned}$$

And in general,

$$u_{n+1} = -(\alpha A - I)u_n, \quad n = 1, 2, \ldots$$

Taking $u_0 = w_0 = 0$, yields:

$$\begin{aligned}
&u_1 = \alpha b \\
&u_2 = -(\alpha A - I)u_1 = -(\alpha A - I)(\alpha b), \\
&u_3 = -(\alpha A - I)u_2 = (\alpha A - I)^2(\alpha b), \\
&\vdots \\
&u_{n+1} = -(\alpha A - I)u_n = (-1)^n(\alpha A - I)^n(\alpha b)
\end{aligned}$$

Therefore the solution can be of the form,

$$x = \sum_{i=0}^{\infty}(-1)^i(\alpha A - I)^i(\alpha b). \tag{2.7}$$

The convergence of the series (2.7), when $\alpha = 1$ is proved for diagonally dominant matrix A (see [2]).

3. Counter example

In this section, we point out an error in an algorithm that appeared in [2]. Consider the following linear programming with m constraints and n variables;

$$\begin{aligned}
&Min \quad z = C\mathbf{x}, \\
&s.t: \quad A\mathbf{x} = \mathbf{b}, \\
&\qquad\quad \mathbf{x} \geqslant 0,
\end{aligned} \tag{3.1}$$

where $C = [c_j]$ is a n vector and $rank(A,b) = rank(A) = m$.