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Comparing and ranking fuzzy numbers using ideal solutions

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ABSTRACT

This paper presents a new approach for comparing and ranking fuzzy numbers in a simple manner in decision making under uncertainty. The concept of ideal solutions is sensibly used, and a distance-based similarity measure between fuzzy numbers is appropriately adopted for effectively determining the overall performance of each fuzzy number in comparing and ranking fuzzy numbers. As a result, all the available information characterizing a fuzzy number is fully utilized, and both the absolute position and the relative position of fuzzy numbers are adequately considered, resulted in consistent rankings being produced in comparing and ranking fuzzy numbers. The approach is computationally simple and its underlying concepts are logically sound and comprehensible. A comparative study is conducted on the benchmark cases in the literature that shows the proposed approach compares favorably with other approaches examined.

1. Introduction

Fuzzy numbers are a convenient concept for adequately representing and arithmetically manipulating imprecise numerical quantities and subjective preferences of decision makers in various decision making situations [1–7]. They are widely used in many applications for solving practical problems of various kinds in real world settings due to the presence of subjectiveness and imprecision inherent in the human decision making process [8–12]. These applications often lead to the comparison and ranking of fuzzy numbers for effective decision making in specific circumstances. As a result, comparing and ranking fuzzy numbers becomes a critical problem to be adequately solved in decision making under uncertainty in real world situations [13–16].

Comparing and ranking fuzzy numbers for determining their overall ranking in a given situation is complex and challenging [4,17–19]. This is because fuzzy numbers usually represented by the possibility distribution [3,20] often overlap each other in many practical situations [4,21]. It is difficult to clearly determine whether one fuzzy number is larger or smaller than another in a given situation, in particular when these two fuzzy numbers are similar [22,23].

There are numerous approaches that have been developed for comparing and ranking fuzzy numbers in the literature. Bortolan and Degani [24], Lee and Li [25], Tseng and Klein [26], Chen and Hwang [21], Chen and Lu [27], Chen and Lu [13], Yeh and Deng [4], Deng [23], and Brunelli and Mezei [28] give an intensive review of existing approaches from different perspectives. These approaches usually can be classified into four major categories, including (a) independent ranking, (b) reference based ranking, and (c) pairwise comparison based ranking, and (d) linguistic approximation [4]. They are developed from different perspectives for comparing and ranking fuzzy numbers with numerous applications for solving various kinds of practical problems in real world settings.

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Existing approaches have shown their applicability in comparing and ranking fuzzy numbers for effectively solving various decision making problems with respect to specific characteristics of individual problems [21,3,5,29,30,9,12]. These approaches, however, are not totally satisfactory in comparing and ranking fuzzy numbers for effective decision making under uncertainty. This is because existing approaches often suffer from some common drawbacks in comparing and ranking fuzzy numbers including the lack of discrimination in differentiating similar fuzzy numbers, the sophistication of individual approaches, the inconsistent and often counter-intuitive ranking outcomes under circumstances, and the considerable computational effort required in specific situations [21,3,4,23,28].

This paper presents an ideal solution based approach for comparing and ranking fuzzy numbers in a simple manner in decision making under uncertainty. The concept of ideal solutions including the positive ideal solution and the negative ideal solution is sensibly used, and a distance-based similarity measure between fuzzy numbers is appropriately adopted for effectively determining the overall performance index value for each fuzzy number in comparing and ranking fuzzy numbers. As a result, all the available information characterizing a fuzzy number is fully utilized, and both the relative position and the absolute position of the fuzzy numbers are appropriately considered. This ensures that consistent ranking outcomes can be obtained in comparing and ranking fuzzy numbers, leading to effective decisions being made in practical decision making situations. The approach is computationally simple, and its underlying concepts are logically sound and comprehensible. A comparative study is conducted on the benchmark cases in the literature that shows the proposed approach compares favorably with other representative approaches in comparing and ranking fuzzy numbers.

In what follows, we first review existing ideal solution based approaches to comparing and ranking fuzzy numbers in the literature. We then present a new approach for comparing and ranking fuzzy numbers. Finally, we present a comparative study for illustrating the applicability of the proposed approach in comparing and ranking fuzzy numbers.

2. Ideal solution based approaches to comparing and ranking fuzzy numbers

A fuzzy number is a convex fuzzy set [3,20,31], characterized by a given interval of real numbers in the real line R, each with a grade of membership between 0 and 1, and defined as follows:

Definition 1. A fuzzy number A in a universe of discourse R is characterized by a membership function $\mu_A(x)$ which associates with each element x in R a real number in the interval [0,1]. The function value $\mu_A(x)$ is commonly referred to as the grade of membership of x in fuzzy number A.

Definition 2. Let A_i ($i = 1, 2, \dots, n$) be a real fuzzy number of the universe of discourse R where

(a) A_i is normal, iff $\sup_{x \in R} \mu_{A_i}(x) = 1$.
(b) A_i is convex, iff $\mu_{A_i}(\lambda x + (1-\lambda)y \geqslant (\mu_{A_i}(x) \wedge \mu_{A_i}(y)),\ \forall x, y \in R, \lambda \in [0,\ 1]$ where the symbol ^ denotes the minimum operator.

Definition 3. If A_i ($i = 1, 2, \dots, n$) is a fuzzy number in which $\{x \geqslant 0,\ \mu_{A_i}(x) \geqslant \alpha, \alpha \in [0,\ 1]\}$, A_i is called a positive fuzzy number.

Definition 4. Let A_i ($i = 1, 2, \dots, n$) be a normal and convex fuzzy number of the universe of discourse R with a piecewise continuous membership function $\mu_{A_i}(x)$

(a) The support of A_i represented by $\{x,\ \mu_{A_i}(x) > 0\}$ is a crisp set.
(b) The α set of A_i ($i = 1, 2, \dots, n$) is a crisp set given by $\{x,\ \mu_{A_i}(x) \geqslant \alpha, 1 \succ \alpha \succ 0\}$.

Definition 5. A fuzzy number A_i ($i = 1, 2, \dots, n$) of the universe of discourse R with a piecewise continuous membership function $\mu_{A_i}(x)$ can be defined [4,14,32] as

$$\mu_{A_i}(x) = \begin{cases} \mu_{A_i}(x), & a_{1i} \leqslant x \leqslant a_{2i}, \\ 1, & a_{2i} \leqslant x \leqslant a_{3i}, \\ \mu_{A_i}(x), & a_{3i} \leqslant x \leqslant a_{4i}, \\ 0, & \text{otherwise}. \end{cases}$$

where $\mu^L_{A_i}(x)$ is the left membership function that is an increasing function, and $\mu^L_{A_i}(x)$: $[a_{1i}, a_{2i}] \to [0, 1]$. $\mu^R_{A_i}(x)$ is the right membership function that is a decreasing function and $\mu^R_{A_i}(x)$: $[a_{3i}, a_{4i}] \to [0, 1]$. In addition, a trapezoidal fuzzy number is usually denoted as (a_1, a_2, a_3, a_4). Often, (a_1, a_2, a_3, a_4) can also signify a triangular fuzzy number (a_1, a_3, a_4) if $a_2 = a_3$ defined as follows: