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Possibility linear programming with trapezoidal fuzzy numbers

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ABSTRACT

Fuzzy linear programming with trapezoidal fuzzy numbers (TrFNs) is considered and a new method is developed to solve it. In this method, TrFNs are used to capture imprecise or uncertain information for the imprecise objective coefficients and/or the imprecise technological coefficients and/or available resources. The auxiliary multi-objective programming is constructed to solve the corresponding possibility linear programming with TrFNs. The auxiliary multi-objective programming involves four objectives: minimizing the left spread, maximizing the right spread, maximizing the left endpoint of the mode and maximizing the middle point of the mode. Three approaches are proposed to solve the constructed auxiliary multi-objective programming, including optimistic approach, pessimistic approach and linear sum approach based on membership function. An investment example and a transportation problem are presented to demonstrate the implementation process of this method. The comparison analysis shows that the fuzzy linear programming with TrFNs developed in this paper generalizes the possibility linear programming with triangular fuzzy numbers.

1. Introduction

In the transportation problems, sometimes the unit cost of transportation may be expressed by fuzzy number [1] other than the real number because of the influence by the various subjective and objective factors. Consequently, the optimization model minimizing the total cost is constructed as a fuzzy mathematical programming. In the investment problem, the interest incomes of money market accounts are changed with market and may not be precise as time progresses. The optimization model maximizing the total net worth is also formulated as a fuzzy mathematical programming [2]. Therefore, the fuzzy mathematical programming is of a great importance for scientific researches and real applications.

The research on fuzzy mathematical programming has been an active area since Bellman and Zadeh [1] proposed the definition of fuzzy decision making [2–21]. The fuzzy mathematical programming was widely applied to many fields, such as transportation plan [12,13], supply chain network [15,16], dynamic virtual hub location [17], investment problem [2]. All these fuzzy mathematical (or linear) programming models can be classified into the following four categories according to the types of the fuzzy numbers.

The first is fuzzy programming model with intervals. For example, Ishibuchi and Tanaka [20] proposed the multi-objective programming in optimization of the interval objective function. They gave the definitions of the maximization and minimization problems with the interval objective functions. The second is fuzzy linear programming model with

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triangular fuzzy numbers (TFNs) [2,18,19]. For example, Lai and Hwang [2] developed a new approach to some possibilistic linear programming problems with TFNs. They transformed the fuzzy linear programming into a multi-objective linear programming model, involving three objective functions: minimizing the low loss, maximizing the most possible value and maximizing the upper the profit. The third is fuzzy linear programming model with trapezoidal fuzzy numbers (TrFNs) [5–11]. For example, Ganesan and Veeramani [8] and Ebrahimnejad [11] studied the fuzzy linear programs with TrFNs, but the constructed fuzzy linear programming models are only suitable for the symmetrical TrFNs. Maleki et al. [5–7], Allahviranloo et al. [9] and Liu [10] utilized the ranking function to solve the fuzzy linear programming models with TrFNs. The last is the optimization in an intuitionistic fuzzy environment [12,13]. For example, Angelove [12] applied the degrees of rejection of constraints and values of the objective to formulate the optimization problem in an intuitionistic fuzzy environment and transformed into the crisp optimization problem. Dubey et al. [13] proposed optimistic approach, pessimistic approach and mixed approach to solve the intuitionistic fuzzy linear programming problem.

Though fuzzy linear programming models with TrFNs have been researched in [5–11], the fuzzy linear programming models studied in [8,11] are only suitable for the symmetrical TrFNs, which significantly restricts the application scope of the models. In [5–7,9,10], the different ranking methods of TrFNs may result in the different crisp linear programming models, thus the obtained optimal solutions may be changed with the chosen ranking methods of TrFNs. Therefore, it is necessary to develop a reliable and stable method to solve the fuzzy linear programming models with TrFNs.

Since TrFN permits two parameters to represent the most possible values, while TFN uses the single parameter to represent the most possible value, TFN is a special case of TrFN. Therefore, TrFN is valuable both for modeling imprecision and for its ability to easily reflect the ambiguous nature of subjective judgments. The aim of this paper is to extend the possibility linear programming with TFNs [2] to develop a new possibility linear programming with TrFNs. In this method, TrFNs are used to capture imprecise or uncertain information for the imprecise objective coefficients and/or the imprecise technological coefficients and/or resources. The auxiliary multi-objective programming is constructed to solve the corresponding possibility linear programming with TrFNs. The auxiliary multi-objective programming involves four objectives: minimizing the left spread, maximizing the right spread, maximizing the left endpoint of the mode and maximizing the middle point of the mode.

We note that a similar idea can be found in the possibility linear programming method introduced by Lai and Hwang [2]. However, significant differences in features exist between the two developed methodologies. First, Lai and Hwang [2] studied the possibility linear programming with TFNs, i.e., the objective coefficients, technological coefficients, and available resources are TFNs other than TrFNs. In contrast, this paper utilizes TrFNs to represent these imprecise variables and proposed the method for solving the possibility linear programming with TrFNs. Since TFNs are a special case of TrFNs, the possibility linear programming with TFNs proposed by Lai and Hwang [2] is just a special case of the one with TrFNs proposed in this paper. Second, Lai and Hwang [2] only adapted the Zimmermam's fuzzy programming method [21] to solve the multi-objective linear programming, while this paper proposes three kinds of approaches to solving the constructed auxiliary multi-objective programming, i.e., the pessimistic approach, optimistic approach, and linear sum approach based on membership function. These approaches greatly enhance the flexibility of the proposed method for different decision makers (DMs). Third, to construct the membership functions of the objective functions, Lai and Hwang [2] need solve six linear programming models to obtain the positive and negative ideal solutions. Conversely, in this paper the maximum objective values can be obtained by solving four linear programming models and thereby the minimum objective values are determined by directly comparing the objective values. Thus, the proposed method in this paper need less computation cost than the method in [2].

The paper is organized as follows. In Section 2, the definition of TrFN is defined and the interval objective programming is introduced. In Section 3, the possibility linear programming with TrFNs is developed. The proposed possibility linear programming method is illustrated with an investment problem and comparison analysis is conducted in Section 4. A potential application to transportation problem is give in Section 5. Concluding remark is given in Section 6.

2. Definition for trapezoidal fuzzy numbers and interval objective programming

2.1. Definition for trapezoidal fuzzy numbers

A fuzzy number $\tilde{m}$ is a special fuzzy subset on the set R of real numbers. Let $\tilde{m} = (l, m_1, m_2, r)$ be a TrFN, where the membership function $\mu_{\tilde{m}}$ of $\tilde{m}$ is

$$\mu_{\tilde{m}}(x) = \begin{cases} \frac{x-l}{m_1-l} & (l \leqslant x < m_1), \\ 1 & (m_1 \leqslant x \leqslant m_2), \\ \frac{r-x}{r-m_2} & (m_2 < x \leqslant r). \end{cases}$$

The closed interval $[m_1, m_2]$ is the mode of $\tilde{m}$. l and r are the lower and upper limits of $\tilde{m}$ [22].

It is easily seen that a TrFN $\tilde{m} = (l, m_1, m_2, r)$ is reduced to a real number m if $l = m_1 = m_2 = r$. Conversely, a real number m can be written as a TrFN $\tilde{m} = (m, m, m, m)$. A TrFN $\tilde{m} = (l, m_1, m_2, r)$ is reduced to a TrFN $\tilde{m} = (l, m_1, r)$ if $m_1 = m_2$.

TrFN $\tilde{m} = (l, m_1, m_2, r)$ is called a positive TrFN if $l \geqslant 0$ and one of l, m_1, m_2 and r is non-zero. Furthermore, $\tilde{m} = (l, m_1, m_2, r)$ is called a normalized positive TrFN if it is a positive TrFN, $l \geqslant 0$ and $r \leqslant 1$.