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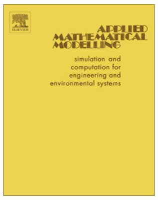

Numerical simulation of sphere water entry problem using Eulerian–Lagrangian method

M. Ahmadzadeh [a],*, B. Saranjam [a], A. Hoseini Fard [a], A.R. Binesh [b]

[a] *Air Naval Research Center, Shiraz, Iran*
[b] *Mechanical Engineering School, Shiraz University, Shiraz, Iran*

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ABSTRACT

This paper looks at the hydrodynamic's numerical simulation of a free-falling sphere impacting the free surface of water by using the coupled Eulerian–Lagrangian (CEL) formulation included in the commercial software ABAQUS. A 3D model of a sphere with an unsteady viscous transient flow condition is used for numerical simulation. The simulation is performed for sphere with different density. The simulation results are verified by showing the computed shape of the air cavity, displacement of sphere, pinch-off time and depth that agree well with experimental results.

1. Introduction

When a solid object strikes a water surface, it may create an air cavity whose form influences the object's subsequent trajectory. Accurate models of this phenomenon are essential for the effective design of air-to-sea projectiles as may be used to target underwater mines and torpedoes [1].

There are many research works on the water entry problems, the experimental data or computational results for low-density floating sphere are very rare. Worthington & Cole used single-spark photography to examine the air cavity formed by the vertical entry of spheres into water and so initiated the scientific investigation of solid–liquid impacts [2,3]. Subsequent studies by Mallock and Bell provided some qualitative explanation for the observed cavity shapes and sphere trajectories [4,5]. Additional investigations of the water-entry cavity and surrounding flow field were performed by Birkhoff & Caywood, Birkhoff & Isaacs, Birkhoff & Zarantonello and Abelson, but the most extensive ones were conducted by May with a view to naval ordinance applications [6–12].

The significance of research on water entry problem is the large damage impact force on ship and ocean structures. In rough sea, large motion of the ocean platforms or ship hulls can usually cause great damage to the structures when they touch and entry the water. This kind of damage sometimes can cause the sink of ocean structure, which will bring huge economic losses caused by hydrodynamic impact loads [13–15]. The complexity of impact of objects on a free surface has piqued the interest of researchers for centuries and remains of interest today. Simulation is commonly used in the many industries to reduce the time and cost associated with experimental tests to build a physical prototyping. Numerical impact simulation of these phenomena is the main objective of this research.

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* Corresponding author. Tel.: +98 711 2262247.
E-mail address: mohsen.ahmadzade@gmail.com (M. Ahmadzadeh).

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When a fluid drop or solid object falls through a free fluid surface, a common occurrence is the appearance of a tall fluid jet rising upwards. This 'Worthington jet' can extend several meters above the surface, depending on the fluid parameters [8]. Typical studies of impact phenomena, whether focusing on the flow above or below the original motionless surface (splash or cavity), have been concerned with high speeds, which translates into high fluid inertia (high *Re*) [8,16].

In recent decades, many methods were developed to solve these kinds of problems. Battistin used a boundary-element formulation with nonlinearities in the free-surface boundary conditions to investigate the water entry of two-dimensional bodies of arbitrary shape. Iafrati solved the flat plate impact problem by using method of matched asymptotic expansions. Kleefsman simulated the water impact problem of falling wedge and cylinder by solving incompressible viscous NS equations with VOF method. Maruzewski developed a SPH method to simulate the water impact process of a snooker ball. A good prediction method is based on the Navier–Stokes equations, which describe the motion of an incompressible, viscous fluid, with a free liquid surface. An overview of the various methods available which describe the motion of an incompressible, viscous fluid, with a free liquid surface can be found in [17–21].

This investigation focused on the problem of a rigid body sphere touching at time $t = 0$ the free surface of a stationary viscous fluid with a variable velocity and finding for $t > 0$, the shape of cavity, the pinch-off time and depth of the rigid sphere by using the coupled Eulerian–Lagrangian formulation. It should be noted that, this formulation is three dimensional and the fluid is assumed to be viscous and compressible. A significant jet flow and the water splash-up are observed in this numerical simulation.

2. Finite element modeling

2.1. Model geometry

Four one-inch diameter spheres, each made from a different material, were used in the present study. Their densities are reported in Table 1. Water density considered is equal to 1000 kg/m^3. The sphere is modeled as shell geometry and a rigid body constraint is subsequently applied to it. Fluid domains are modeled as a cube with 30 cm wide and 50 cm long with a depth capacity of 60 cm.

2.2. Material properties

2.2.1. Energy equation and Hugoniot curve

The equation for conservation of energy equates the increase in internal energy per unit mass, E_m, to the rate at which work is being done by the stresses and the rate at which heat is being added. In the absence of heat conduction the energy equation can be written as;

$$\rho \frac{\partial E_m}{\partial t} = (p - p_{bv})\frac{1}{\rho}\frac{\partial \rho}{\partial t} + S : \dot{e} + \rho \dot{Q},$$

where p is the pressure stress defined as positive in compression, p_{bv} is the pressure stress due to the bulk viscosity, S is the deviator stress tensor, $\dot{e}$ is the deviator part of strain rate, and $\dot{Q}$ is the heat rate per unit mass. The equation of state is assumed for the pressure as a function of the current density, ρ, and the internal energy per unit mass, E_m;

$$p = f(\rho, E_m),$$

which defines all the equilibrium states that can exist in a material. The internal energy can be eliminated from the above equation to obtain a p vs. V relationship (where V is the current volume) or, equivalently, a p vs. $1/\rho$ relationship that is unique to the material described by the equation of state model. This unique relationship is called the Hugoniot curve and is the locus of $p - V$ states achievable behind a shock (Fig. 1).

The Hugoniot pressure, p_H, is a function of density only and can be defined, in general, from fitting experimental data. An equation of state is said to be linear in energy when it can be written in the form;

$$p = f + gE_m,$$

where $f(\rho)$ and $g(\rho)$ are functions of density only and depend on the particular equation of state model.

Table 1
Densities of the spheres used in the study.

Material	Density (kg/m^3)
Steel	7860
Teflon	2300
Nylon	1140
Polypropylene	860