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Large deflection of magneto-electro-elastic laminated plates

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ABSTRACT

A model for the large deflection analysis of magneto-electro-elastic laminated plates is derived. The first order shear deformation theory and the von Karman stress function approach are employed. A set of resolving partial differential equations involving kinematical variables and the stress function is obtained as a consequence of the preliminary condensation of the electro-magnetic state to the plate kinematics. A closed form solution for simply-supported plates is presented. Numerical results are carried out for plates consisting of piezoelectric $BaTiO_3$ and piezomagnetic $CoFe_2O_4$ layers. These results show the influence of large deflections on the plate response and could be useful in the analysis and design of layered magneto-electro-elastic composite plates.

1. Introduction

Smart structures are of interest in many technological fields, including the next generation of transport vehicles, health monitoring, vibration control, sensor and actuator applications, robotics, medical instruments and energy harvesting only to name a few. Indeed, they allow to design and employ multi-functional components as a mean to accomplish high reliability and performance requirements. In this context, smart laminates belong to a very important and promising class of new generation smart structures. In particular, magneto-electro-elastic composites containing piezoelectric and piezomagnetic phases have recently emerged [1] owing to their ability to convert energy among the electric, magnetic and elastic form and also as they exhibit a new property known as magnetoelectric effect, which refers to the capability of passively coupling the electric and magnetic fields [2]. The laminated form of these composites appears more efficient with respect to the bulk form and their modeling is essential to properly understand their behavior and provide for adequate analysis and design.

Due to the multi-layer configuration and the multi-field behavior of the employed materials, structural modeling of smart laminates requires special attention. The most accurate characterization of the smart laminates response is obtained by analytical exact or numerical 3-*D* solutions, when no assumptions are made on the variation of the involved field variables. However, analytical exact solutions can be obtained for specific geometries and boundary conditions only, whereas 3-*D* finite element solutions are carried out with meaningful computational costs. So 2-*D* efficient laminate theories play an important role as they allow to reduce the analysis effort preserving a suitable level of accuracy and providing the basis to develop more efficient numerical solutions.

For the analysis of magneto-electro-elastic laminates Pan [3] presented an exact 3-*D* solution for simply supported plates, which was implemented by Pan and Heyliger [4] for the case of cylindrical bending. The same authors also proposed approximate solutions for two and three-dimensional problems [5,6]. Wang and Shen [7] studied the general solution for three-dimensional problems of transversely isotropic magneto-electro-elastic media by using potential functions. The state

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space formulation was developed by Wang et al. [8]. More recently, Liu [9] proposed the employment of a 2-*D* plate theory for homogeneous single-layer thin plates. Finite elements solutions have been presented by Lage et al. [10], Phoenix et al. [11] and Carrera and Nali [12]; these are generally based on layerwise modelization. Also boundary elements approaches, applicable to plate cylindrical bending, were proposed by Milazzo et al. [13] and Aimin et al. [14].

The literature survey evidences that the magneto-electro-elastic laminate solutions are prevalently obtained by 3-*D* analytical or finite elements layerwise-like theories. These assume the electro-magnetic state quantities as independent unknowns leading to increasing computational effort as the number of layer increases. This prompts for the development of alternative multiphysics 2-*D* plate models, able to reduce the solution effort and providing efficient tools for the design and optimization procedures. In this sense, from the mechanical point of view, equivalent single-layer theories for laminated structures lead to plate models that have a solution complexity independent from the number of layers. Therefore, the extension of the equivalent single-layer theories to magneto-electro-elastic laminates, provided for a possible preliminary analytic condensation of the electro-magnetic state to the mechanical variables, represents a sound underlying idea to develop an efficient smart plate model. Additionally, such a modeling strategy could take advantage of the solution tools available for the mechanics of multi-layered plates. Basing on these observations, the author and co-workers recently proposed the use of equivalent single layer theories for magneto-electro-elastic laminated beams [15,16] and plates [17,18].

All of the above-cited approaches refer to the case of plate small displacements, whereas little attention was devoted to the nonlinear large displacements case, and only very recently Xue et al. [19] presented a large deflection solution for homogeneous single-layer rectangular magneto-electro-elastic thin plates based on the von Karman's plate assumptions. To the best of the author's knowledge, no closed form solutions for large deflection of multilayered magneto-electro-elastic laminates have been proposed notwithstanding the large displacements influence can play important role in practical applications.

Based on this consideration, in the present paper, an equivalent single-layer approach for the large deflection analysis of multilayered magneto-electro-elastic laminates under static loads is proposed for the first time. It is based on the approach previously presented by the author and co-workers for the small displacement case [17,18], which is here novelly formulated for the large deflection case by using the von Karman's plate assumptions and providing a closed form solution for simply-supported magneto-electro-elastic laminated plates. In particular, the first order shear deformation theory is assumed to model the plate deformation considering the non linear strain–displacement relations for large deflection of plates, whereas the Maxwell equations for electrostatics and magnetostatics are used to model the electric and magnetic behavior. The plate model is obtained by providing for a preliminary solution of the electro-magnetic state as a function of the mechanical variables. In turn, taking this result into account, the plate mechanical governing equations, namely equilibrium and compatibility equations, are written leading to a system of coupled differential equations whose solution determines the plate mechanical response in terms of kinematical variables and stress function. Once the mechanical problem is solved the quantities associated with the electro-magnetic behavior can be recovered by simple postprocessing. Some results for typical magneto-electro-elastic simply-supported laminates are obtained by the Galerkin method and presented to show the effect of the large deflection on the plate behavior.

2. Basic equations

Let us consider an *N*-layered, rectangular plate with side lengths L_x and L_y and thickness *h*. The plate is referred to a Cartesian coordinate system with the *x* and *y* coordinates spanning the midplane and the *z*-axis directed along the plate thickness. The layers have constant thickness and are made of homogeneous and orthotropic magneto-electro-elastic materials with electric and magnetic poling direction parallel to the *z*-axis and material principal directions parallel to the *x*- and *y*-axes. Obviously, the case of piezoelectric, magnetoelastic or pure elastic layers is included. The faces of the *k*th layer are located

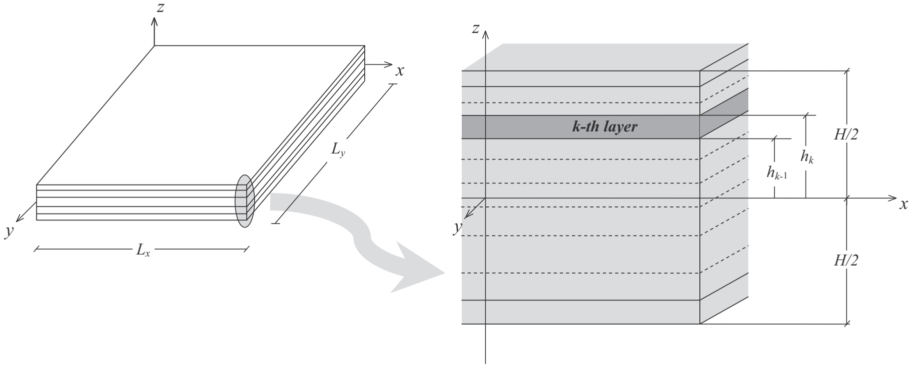

Fig. 1. Laminate geometrical scheme.