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Computation of transmission coefficients in the plain and corrugated electro-magnetic waveguides using finite point set method

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A B S T R A C T

Maxwell's equations are solved in an electro-magnetic rectangular waveguide with the boundary determined by material properties and interface conditions wherein finite point set method approximates the partial derivatives of the scattered fields. The process is time dependent. Waveguide propagation depends on the operating wave length, polarization, shape and aspect ratio of the waveguide.

A continuous incident pulse is used to study the electric field pattern and transmission behavior in plain and corrugated waveguides. The waveguide is modeled in a 2D rectangle with incident source at the left boundary.

Transmission coefficients are computed as a function of frequency of the continuous pulse for both the plain and corrugated waveguide of various heights $\lambda/2$, $\lambda/4$ and $3\lambda/4$.

1. Introduction

Passive components are those that do not require electric power to operate in electrical, computer or storage systems. The study on microwave passive components like transmission lines (coaxial, rectangular and cylindrical waveguides) is essential to characterize transmission behavior and attenuation of the wave in the frequency domain. Computation of the parameters transmission coefficient and attenuation coefficient under various test defect conditions through numerical modeling and then simulation helps better understanding and proper design of microwave circuits.

As the microwave pulse incident at the source of the waveguide, the conducting walls of the waveguide control the electro-magnetic waves interior to it. The waves travel longitudinally down the waveguide till the opposite wall and then reflected back. The process results in a component of either electric field or magnetic field in the direction of propagation of the guided wave. Therefore, the wave is no longer a transverse electromagnetic wave (TEM) [1–4].

Rectangular waveguide is one of the earliest type of transmission lines [5] still commonly used in many current applications. Ratanadecho et al. [6] investigated both numerically and experimentally the heating of a liquid layer by microwave with a rectangular waveguide and showed that the heating kinetic energy depends on dielectric properties. Recently, Erol and Balik [7] introduced 2D finite difference method to analyze widely used rectangular waveguides and validated computed results with analytical solution. Tada et al. [8] employed a two dimensional finite difference time domain method in a partially filled microwave applicator to clearly describe electro-magnetic interferences and power absorption in the dielectric inserted waveguide, operating in TE_{10} mode at a frequency 2.45 GHz. Koshiba et al. [9] studied finite element analysis for electro-magnetic waveguides [10,11] in various approaches capable of suppressing and eliminating the spurious solutions.

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Nomenclature

Ω	computational domain
$x,\ y,\ z$	cartesian coordinates
t	time, s(seconds)
dx	difference between the two points $(= \lambda/20)$, m(meters)
h	radius to find neighborhood points, m
λ	wave length (=c/f), m
f	frequency, Hz
ω	angular frequency, cycles/s
c	velocity of light $(= 3 \times 10^8)$, m/s
Nt	number of time steps
Nt_{max}	maximum number of time steps
np	total number of points
dt	magic time step, $(= \frac{dx}{2*c})$, s
f_s	sampling frequency $(= \frac{1}{dt})$, Hz
ps	picosecond
rad	radius of corrugate in x-*direction,* m
$rad1$	radius of corrugate in y-*direction,* m
$\boldsymbol{\mu}_z$	magnetic permeability, henrys/m
ϵ_z	electrical permittivity, farads/m
μ_r	relative permeability, dimensionless
$\boldsymbol{\epsilon}_r$	relative permittivity, dimensionless
σ	electrical conductivity, siemens/m
ξ_m	equivalent magnetic loss, ohms/m
E_z	electric field in z-direction, volts/m
$H_x,\ H_y$	magnetic field in x-, y-direction, amperes/m

Electro-magnetic guided waves and ultrasonic guided waves propagation through corrugation [12], bends and defects have been studied by many researchers to understand reflection and transmission characteristics at the probes. Demma et al. [13] explained the frequency dependent transmission behavior in terms of modes propagating in the straight and curved sections of the pipe using finite element method. Takahashi et al. [14] investigated a high power, 170 GHz, long pulse RF experiments of ITER relevant transmission line consisting of 63.5 mm circular corrugated waveguides [15] and successfully obtained transmission efficiency 92% from the inlet of the transmission line to the end.

Smoothed particle methods in electro-magnetic wave applications have been studied extensively. Recently, Ala et al. [16] employed pairwise interaction dynamically to solve Maxwell's equations with perfectly matched layer boundary conditions in a general 2D domain. Frequency of excitation used here is 1.8 GHz.

The present paper addresses finite point set solution [17] of Maxwell's equations in a rectangular waveguide with and without corrugation. The actual problem, governing equations and the corresponding boundary conditions are described schematically in Section 2. The developed numerical scheme wherein time derivatives discretization by central finite difference scheme while spatial derivatives approximation by finite point set method are presented systematically in Section 3. Finally, scattering of electric field results and evaluation of transmission coefficients are thoroughly discussed in Section 4.

2. Mathematical model

In order to demonstrate transmission across the waveguide, a hollow rectangular cavity is modeled in a 2D computational domain (Ω). Due to biaxial symmetry of the cavity about the two axes, only one quarter of the cross section of the cavity is considered for the present study. A lossless media is chosen to avoid electric and magnetic field losses inside the waveguide.

Fig. 1 shows the two dimensional corrugated waveguide with the excitation source $E_z - > 1.0 * cos(k\omega dt)$, $k = 1, 2 \ldots, Nt_{max}$, $\omega = 2\pi f$ at $x = 0$. The corrugated plane of the waveguide is divided into two parts: corrugated space and free space. The corrugated space is rectangle in shape, have centers denoted as A (icenter,jcenter) and B (icenter,jcenter1).

A microwave beam of light passes through a waveguide, it generates electric and magnetic fields inside it. In general, microwaves are electro-magnetic waves classified by frequencies.

2.1. Governing equations

The equations which govern the electric and magnetic fields propagation that vary with time in the 2D computational domain indicated by physical laws are the Maxwell's equations. The electric field E and the magnetic field H are vector fields