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The modeling mechanism, extension and optimization of grey GM (1, 1) model

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ABSTRACT

The modeling mechanism of GM (1, 1) model is studied by using the thought of matrix analysis in this paper, the extension form GGM (1, 1) model based on the fractional order accumulated generating is put forward and its theoretical significance is analyzed. Furthermore, the influence of multiple transformation, translation transformation for the initial value and generating series on model parameters and predictive value are researched, then the quantitative relation among them is deduced and an optimization model and corresponding algorithm in practical modeling are presented.

1. Introduction

Grey model is an important branch of grey system theory [1–3], since it was pioneered by professor Deng in 1982, grey models, including classical GM (1, 1) model, GM $(1, 1, \alpha)$ model, phase model, hopping model, generalized accumulated model, strengthening operator model, weakening operator model and so on, have been widely used in industrial and agricultural production, engineering, technology, military affairs, economy and many other fields. However, classical GM (1, 1) model is the foundation of these models. Thus it becomes a critical problem to improve the fitting and forecasting precision of GM (1, 1) model.

In order to increase the model precision, scholars have been researching new modeling technology in practice. The results show that the mode of original data, the manner of accumulated generating and the selection of background value are the main factors which influence the precision of grey model, and thus some methods for improving model precision are presented correspondingly. For original data processing, Li and Dai [4] improved the model predictive precision by modifying the initial value; For oscillatory sequence, Qian and Dang [5] used accelerating translation transformation, then improved smoothness by weighted mean generating and modeling finally; Zeng and Xiao [6] improved original data smoothness by using the power function; Using the buffer operator axiom for shock disturbed data sequence, Dang et al. [7,8] constructed a new class of weakening buffer operator and strengthening buffer operator, and study their range of application, Song and Xiao [9] put forward reverse accumulation generation relative to traditional accumulated generating, and presented grey GOM (1, 1) model which provided a new generating method for grey modeling; Papers [10,11] also researched accumulated generating; Huang and Zhong [12] established matrix form for solving parameters of a generalized accumulated grey prediction control model based on matrix theory; Rao et al. [13,14] established hopping model and phase model for original data with jump point and stage value respectively, and studied the matrix form for solving model parameters; Aiming at the unequal interval data sequence

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existed in the research of ultimate bearing capacity, Peng [15] established generalized unequal interval grey forecasting model, and presented the matrix form for solving model parameters. For background value-building, Tan used numerical method to improve the background value [16], and improved the background value for equal interval sequence and unequal interval sequence [17]; Yang et al. [18] improved the background value selection of traditional GOM (1, 1) model by combining the characteristics of reverse accumulation generation. However, these studies are only analysis and optimization for a certain model in a certain aspect, which have no widespread utility and universal extension.

On the problem of fractional order, Leibniz wrote a letter to L 'Hospital wrote a letter early in 1695, and discussed the fractional calculus and the simplest fractional differential equations. Then the research on fractional order experienced a long and slow term, until the end of twentieth Century, the researchers found that the fractal geometry, the power law phenomenon and the related phenomenon or process of memory process can establish close contact with the fractional calculus, and set off a research upsurge. Literature [19] thought that the actual systems are usually fractional order, and using the fractional order describe object can reveal the essential feature better. Literatures [20] studied the stability of fractional order system and its controllability and objectivity. Recently, Wu [21] established the grey system model based fractional order accumulative generation by applying the fractional order to grey model. This paper studies the grey model of generalized accumulation and put forward the GGM (1, 1) model based on the technology of fractional order accumulation generation. By further researching, it is found that the model is actually the unification of GM (1, 1) model, stepping model, hopping model, unequal interval model and so on. Combined with the boundary conditions, the model is optimized by considering grey generating ways. We study the influence of model parameters and predictive value by the translation transformation of generating series and deduce the quantitative relation of model parameters and predictive value though and not through transformation. On the basis of the relation, we establish the optimized model and solve the optimization transformation value, then give it in practical application.

2. The modeling mechanism of GM (1, 1)

Let $x^{(0)}$ be raw series, $x^{(1)}$ is said to be the one order accumulated generating operation series of $x^{(0)}$ namely $x^{(1)} = AGOx^{(0)}$, and $z^{(1)}$ is said to be MEAN series of $x^{(1)}$, that is $z^{(1)} = MEANx^{(1)}$, where

$$x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)),$$

$$x^{(1)}(k) = \sum_{m=1}^{k} x^{(0)}(m),$$

$$x^{(1)} = (x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)),$$

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1),$$

$$z^{(1)} = (z^{(1)}(1), z^{(1)}(2), \ldots, z^{(1)}(n)),$$

$$\forall z^{(1)}(k) \in z^{(1)} \Rightarrow k \in K' = \{2, 3, \ldots, n\}, \quad n \geqslant 4,$$

$$\forall x^{(0)}(k) \in x^{(0)} \Rightarrow k \in K' = \{1, 2, 3, \ldots, n\}, \quad n \geqslant 4.$$

The definition type of GM (1, 1) model is given as follows

$$GM(1,1,D): x^{(0)}(k) + az^{(1)}(k) = b,$$

where a is called the development coefficient and represents the development state of the prediction value, b is called the grey action quantity and represents change contained in the data, (a, b) is the first order parameter bag of GM (1, 1) model and is denoted by symbol P_{I} which can be expressed as vector or sequence, that is,

$$P_{\mathrm{I}} = (a, b)^T.$$

By least-square method the matrixes formulate of parameters bag P_{I} is given as follows

$$P_{\mathrm{I}} = (a, b)^T = (B^T B)^{-1} B^T X,$$

where

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}$$