Applied Mathematical Modelling 38 (2014) 5217–5230

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Generalized magneto-thermoelasticity with two temperature and initial stress under Green–Naghdi theory

Sarhan Y. Atwa *

Higher Institute of Engineering, Department of Engineering Mathematics and Physics, Shorouk Academy, Egypt

ARTICLE INFO

Article history:
Received 19 May 2012
Received in revised form 22 March 2014
Accepted 7 April 2014
Available online 21 April 2014

Keywords:
Generalized thermoelasticity
Hydrostatic initial stress
Two temperature
Green–Naghdi theory

ABSTRACT

Magneto-thermoelastic interactions in an initially stressed isotropic homogeneous elastic half-space with two temperature are studied using mathematical methods under the purview of the Green–Naghdi theory with type II and III. The medium is considered to be permeated by a uniform magnetic field. The normal mode analysis is used to obtain the exact expressions for the displacement components, force stresses, temperature and couple stresses distribution. The variations of the considered variables through the horizontal distance are illustrated graphically. Comparisons are made with the results between type II and III. Numerical work is also performed for a suitable material with the aim of illustrating the results.

1. Introduction

The classical theory of thermoelasticity as exposed, for example, in Carlson's article [1] has been generalized and modified into various thermoelastic models that run under the label of hyperbolic thermoelasticity (see the survey of Chandrasekharaiah, [2] and Hetnarski and Ignazack, [3]). Lord and Shulman (L–S) [4] introduced the theory of generalized thermoelasticity with one relaxation time by postulating a new law of heat conduction to replace the classical Fourier law. This new law contains the flux vector as well as its time derivative. It contains also a new constant that acts as a relaxation time. The heat equation of this theory is of the wave-type, ensuring finite speeds of propagation for heat and elastic waves. The remaining governing equations for this theory, namely, the equations of motions and the constitutive relations, remain the same as those in the coupled and the uncoupled theories. Müller [5] first introduced the theory of generalized thermoelasticity with two relaxation times. A more explicit version was then introduced by Green and Laws [6], Green and Lindsay [7] and independently by Suhubi [8]. In this theory the temperature rates are considered among the constitutive variables. This theory also predicts finite speeds propagation as in the (L–S) theory. It differs from the latter in that Fourier's law of heat conduction is not violated if the body under consideration has a center of symmetry. The notation hyperbolic reflects the fact that thermal waves are modeled, avoiding the physical paradox of the infinite propagation speed of the classical model. In the 1990s Green and Naghdi [9–11] proposed three new thermoelastic theories based on an entropy equality rather than the usual entropy inequality. The constitutive assumptions for the heat flux vector are different in each theory. Thus, they obtained three theories they called thermoelasticity of type I, thermoelasticity of type II and thermoelasticity of type III. When the type I theory is linearized we obtain the classical system of thermo-elasticity. The type II theory (is a limiting case of type III) does not admit energy dissipation.

* Address: Higher Institute of Engineering, Department of Engineering Mathematics and Physics, Shorouk Academy, Egypt.
E-mail address: srhan_1@yahoo.com

http://dx.doi.org/10.1016/j.apm.2014.04.023
0307-904X/

Nomenclature

λ, μ	counterparts of Lame's parameters
U	specific internal energy
h_i	components of the heat flux vector
P	initial pressure
α	the volume coefficient of thermal expansion
k_T	the isothermal compressibility
δ_{ij}	Kronecker delta
ρ	mass density
C_E	specific heat at constant strain
$K(\geqslant 0)$	thermal conductivity
K^*	material characteristic of theory
θ	temperature distribution
ϕ	conductive temperature
b	two temperature parameter
α_t	coefficient of linear thermal expansion
σ_{ij}	components of the stress tensor
u_i	components of the displacement vector
e_{ij}	components of the strain tensor
e	cubical dilatation
μ_0	magnetic permeability
ε_0	electric permittivity
F_i	Lorentz force
J	the current density vector

Chen and Gurtin [12] and Chen et al. [13,14] formulated a theory of heat conduction in deformable bodies, which depends upon two distinct temperature, the conductive temperature χ and the thermodynamic temperature θ. The two-temperature theory involves a material parameter $\chi \succ 0$. The limit $\chi \to 0$ implies that $\chi \to \theta$ which indicates that the difference between these two temperatures is proportional to the heat supply, and in the absence of any heat supply, the two temperature are identical and the classical theory can be recovered from the two-temperature theory. The two-temperature model has been widely used to predict the electron and photon temperature distributions in ultra-short laser processing of metals. Warren and Chen [15] stated that these two temperatures can be equal in time-dependent problems under certain conditions, whereas χ and θ are generally different in particular problems involving wave propagation. Following Boley and Tolins [16], they studied the wave propagation in the two-temperature theory of coupled thermoelasticity. They showed that the two temperatures θ and χ, and the strain are represented in the form of a traveling wave plus a response, which occurs instantaneously throughout the body. Puri and Jordan [17] discussed the propagation of harmonic plane waves in the two-temperature theory. Quintanilla and Jordan [18] presented exact solutions of two initial-boundary value problems in the two-temperature theory with dual-phase-lag delay. Youssef [19] formulated a theory of two-temperature generalized thermoelasticity. Kumar and Mukhopadhyay [20] extended the work of Puri and Jordan [17] in the context of the linear theory of two-temperature generalized thermoelasticity formulated by Youssef [19]. Recently, Youssef [21] presented a theory of two-temperature thermoelasticity without energy dissipation.

The development of initial stresses in the medium is due to many reasons, for example resulting from differences of temperature, process of quenching, shot pinning and cold working, slow process of creep, differential external forces, gravity variations, etc. The earth is assumed to be under high initial stresses. It is therefore of much interest to study the influence of these stresses on the propagation of stress waves. Biot [22] showed the acoustic propagation under initial stress which is fundamentally different from that under stress-free state. He has obtained the velocities of longitudinal and transverse waves along the co-ordinate axis only. The wave propagation in solids under initial stresses has been studied by many authors for various models. Montanaro [23] investigated the isotropic linear thermoelasticity with a hydrostatic initial stress. Singh [24] studied the effect of hydrostatic initial stresses on waves in a thermoelastic solid half-space.

Othman et al. [25] studied the effect of thermoelastic Plane Waves for an Elastic Solid Half-space under Hydrostatic Initial Stress of Type III. Othman and Sarhan [26] studied the effect of magnetic field on 2-D problem of generalized thermoelasticity without energy dissipation. Ailawalia et al. [27] discussed the effect of rotation due to various sources at the interface of elastic half space and generalized thermoelastic half space. Othman et al. [28] studied the effect of diffusion on the two-dimensional problem of generalized thermo-elasticity with Green–Naghdi theory.

Investigation of the interaction between the magnetic field and stress and strain in a thermoelastic solid is very important due to its many applications in the field of geophysics, plasma physics and related topics, especially in the nuclear field,