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Scheduling deteriorating jobs with a learning effect on unrelated parallel machines

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ABSTRACT

In this study we consider unrelated parallel machines scheduling problems with learning effect and deteriorating jobs, in which the actual processing time of a job is a function of joint time-dependent deterioration and position-dependent learning. The objective is to determine the jobs assigned to corresponding each machine and the corresponding optimal schedule to minimize a cost function containing total completion (waiting) time, total absolute differences in completion (waiting) times and total machine load. If the number of machines is a given constant, we show that the problems can be solved in polynomial time under the time-dependent deterioration and position-dependent learning model.

1. Introduction

In traditional scheduling problems and models, it is assumed that the processing time of a job is a constant value. However, there are many real situations where the job processing time may be subject to change due to deterioration jobs and/or learning effect phenomena. For example, when the processing times arise from manual operations, the possibility of learning exists. On the other hand, it has been noticed that jobs may deteriorate as they wait to be processed, e.g., in steel production, emergency medicine, national defense or cleaning assignments, where any delay in starting to process a job increases the time for accomplishing the job. Extensive surveys of different scheduling models and problems involving deteriorating jobs can be found in Alidaee and Womer [1], and Cheng et al. [2]. Gawiejnowicz [3] recently presented an updated survey of the results on scheduling problems with deteriorating jobs and time-dependent processing times. An extensive survey of different scheduling models and problems with learning effects and/or deterioration jobs could be found in Biskup [4]. More recent papers which have considered scheduling jobs with learning effects and/or deterioration effects include Wu et al. [5], Oron [6], Mosheiov [7], Barketau et al. [8], Cheng et al. [9], Cheng et al. [10], Lee and Wu [11], Wu and Lee [12], Janiak et al. [13], Lee et al. [14], Li et al. [15], Lee et al. [16], Ji and Cheng [17], Huang and Wang [18], Wang et al. [19], Lee [20,21], Lee and Lai [22], Cheng et al. [23], Mosheiov [24], Lee et al. [25], Wang et al. [26], Wang et al. [27], Bai et al. [28], Wang et al. [29], Wang et al. [30], Lai and Lee [31], Lee and Chung [32], Lee [33], and Huang et al. [34].

Most of the scheduling problems considers regular objective functions, i.e., an objective function which is nondecreasing with respect to job completion times (e.g., the total completion time TC=$\sum_{j=1}^{n} C_j$, the total waiting time TW=$\sum_{j=1}^{n} W_j$, the makespan $C_{\max} = \max\{C_j | j = 1, 2, \ldots, n\}$, where C_j (W_j) is the completion (waiting) time of job J_j, and n is the number of

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jobs). However, there are many applications in which non-regular objective function. For instance, in any service or manufacturing setting, one might be interested in providing much uniform quality of service based on the jobs's (customers') waiting times in system (Merten and Muller [35]). Merten and Muller [35], Schrage [36], Eilon and Chowdhury [37], Vani and Raghavachari [38], Kubiak [39], and Srirangacharyulu and Srinivasan [40] considered completion time variance (i.e., $CTV = \frac{1}{n}\sum_{j=1}^{n}(C_j - \overline{C})^2$, where $\overline{C} = \frac{1}{n}\sum_{j=1}^{n} C_j$) and/or waiting time variance (i.e., $WTV = \frac{1}{n}\sum_{j=1}^{n}(W_j - \overline{W})^2$, where $\overline{W} = \frac{1}{n}\sum_{j=1}^{n} W_j$) problems. They established several properties of the optimal schedules for these measures, and proposed heuristic algorithms. Kanet [41] considered the total absolute differences in completion times (i.e., $TADC = \sum_{k=1}^{n}\sum_{j=k}^{n}|C_k - C_j|$) minimization problem, he proved that this problem can be solved in polynomial time. Bagchi [42] studied the total absolute differences in waiting times (i.e., $TADW = \sum_{k=1}^{n}\sum_{j=k}^{n}|W_k - W_j|$) minimization problem, he proved that TADW minimization can be solved in polynomial time. Wang and Xia [43] considered single machine scheduling with controllable processing times. The objective function is to minimize a cost function containing TC (TW), TADC (TADW) and total compression cost. They solved the problem by formulating it as an assignment problem. Oron [6] considered a single machine TADC minimization scheduling with simple linear deterioration. They proved some properties of an optimal schedule, and introduced two heuristic algorithms to solve this problem. Li et al. [15] considered a single machine TADC minimization scheduling with proportional linear deterioration. Huang and Wang [18] considered parallel identical machines scheduling problems with deteriorating jobs. They proved that the TADC (TDAW) minimization problem can solvable in polynomial time. Huang et al. [34] considered parallel identical machines scheduling problems with deteriorating jobs and learning effect. For minimizing a cost function containing TC (TW) and TADC (TADW), they gave an optimal polynomial time algorithm.

Recently, there is growing interest in scheduling research that considers deteriorating jobs and learning effects at the same time [9,22,26–31,33,34]. The phenomena of learning effect and deteriorating jobs occurring simultaneously can be found in the manufacturing environment, in order to provide customers with greater product variety, organizations are moving towards shorter production runs and frequent product changes. The learning and forgetting that workers undergo in this environment have thus become increasingly important as workers tend to spend more time in rotating among tasks and responsibilities prior to becoming fully proficient. These workers are often interrupted by product and process changes causing deterioration in performance, which we will refer to, for simplicity, as forgetting (Nembhard and Osothsilp [44], and Wang [45]). Another example in which silicon-based raw material is first heated up until it becomes a lump of malleable dough from which the craftsman cuts pieces and shapes them according to different designs into different glass craft products. On the other hand, the pieces that are shaped later require shorter shaping times because the craftsman's productivity improves as a result of learning (Cheng et al. [9], Lee [33]).

However, most of the research with deteriorating jobs and learning effect considers the single machine case, in fact, the parallel machine case is interesting and closer to real problems. In this paper, we study unrelated parallel machines scheduling problems with learning effect and deteriorating jobs. The objective is to minimize the weighted sum of total completion (waiting) time, total absolute differences in completion (waiting) times and total machine load.

In the next section we briefly formulate the model. In Section 3 we study unrelated parallel machines scheduling problems. Finally, we present the conclusions.

2. Problems description

We consider n independent jobs $J = \{J_1, J_2, \ldots, J_n\}$ which need to be processed on m unrelated parallel machines $\{M_1, M_2, \ldots, M_m\}$ such that all jobs and machines are available at time 0. The machine can handle one job at a time and preemption is not allowed. Let n_i denote the number of jobs assigned to M_i and $P_m^n = (n_1, n_2, \ldots, n_m)$ denote a job-allocation vector, where $\sum_{i=1}^{m} n_i = n$. We assume, as in most practical situations, that $m < n$. Associated with each job J_j $(j = 1, 2, \ldots, n)$ on machine M_i there is a normal (basic) processing time $a_{ij} \geqslant 0$. As in Wang [45], we assume that the actual processing time of job J_j on machine M_i if it is scheduled in position r in a sequence is given by:

$$p_{ij} = (a_{ij} + bt)r^a, \tag{1}$$

where $t \geqslant 0$ is the starting time for job J_j on machine M_i, $a \leqslant 0$ is the learning index, given as the (base 2) logarithm of the learning rate, and $b \geqslant 0$ is the common deterioration rate for all jobs.

For a given sequence $\pi = [J_1, J_2, \ldots, J_n]$, $C_{ij} = C_{ij}(\pi)$ represents the completion time for job J_j on machine M_i and $W_{ij} = C_{ij} - p_{ij}$ represents the waiting time of job J_j on machine M_i. Let $TADC = \sum_{i=1}^{m}\sum_{k=1}^{n_i}\sum_{j=k}^{n_i}|C_{ik} - C_{ij}|$, $TADW = \sum_{i=1}^{m}\sum_{k=1}^{n_i}\sum_{j=k}^{n_i}|W_{ik} - W_{ij}|$, $TC = \sum_{i=1}^{m}\sum_{j=1}^{n_i} C_{ij}$, $TW = \sum_{i=1}^{m}\sum_{j=1}^{n_i} W_{ij}$, $TML = \sum_{i=1}^{m} C_{\max}^i$, be the total absolute differences in completion times, the total absolute differences in waiting times, the total completion times, the total waiting times and the total machine load, respectively, where $C_{\max}^i$ denotes the makespan of machine M_i. The goal is to determine the jobs assigned to corresponding each machine and the corresponding optimal schedule so that the corresponding value of the following cost functions be optimal:

$$f_1(\pi) = \delta_1 TC + \delta_2 TADC + \delta_3 TML, \tag{2}$$