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Finite scale Lyapunov analysis of temperature fluctuations in homogeneous isotropic turbulence

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ABSTRACT

This study analyzes the temperature fluctuations in incompressible homogeneous isotropic turbulence through the finite scale Lyapunov analysis of the relative motion between two fluid particles. The analysis provides an explanation of the mechanism of the thermal energy cascade, leads to the closure of the Corrsin equation, and describes the statistics of the longitudinal temperature derivative through the Lyapunov theory of the local deformation and the thermal energy equation. The results here obtained show that, in the case of self-similarity, the temperature spectrum exhibits the scaling laws κ^n, with $n \approx -5/3, \ -1$ and $-17/3 \div -11/3$ depending upon the flow regime. These results are in agreement with the theoretical arguments of Obukhov–Corrsin and Batchelor and with the numerical simulations and experiments known from the literature. The PDF of the longitudinal temperature derivative is found to be a non-gaussian distribution function with null skewness, whose intermittency rises with the Taylor scale Péclet number. This study applies also to any passive scalar which exhibits diffusivity.

1. Introduction

This work adopts the finite-scale Lyapunov theory for studying the temperature fluctuations in incompressible homogeneous isotropic turbulence in an infinite fluid domain. The study is mainly motivated by the fact that, in isotropic turbulence, the temperature spectrum $\Theta(\kappa)$ exhibits several scaling laws κ^n in the different wavelength ranges depending on R and Pr [1–4], where R and Pr are Taylor scale Reynolds number and Prandtl number, respectively. This is due to the peculiar connection between temperature fluctuations, fluid deformation and velocity field, whose effect varies following R and Pr.

For large values of R and Pr, [1,2] argumented, through the dimensional analysis, that $\Theta(\kappa) \approx \kappa^{-5/3}$ in the so-called inertial-convective subrange (see Fig. 1). Batchelor [3] considered the isotropic turbulence at high Prandtl number, when R is assigned. There, the author assumed that, at distances less than the Kolmogorov scale, the temperature fluctuations are mainly related to the strain rate associated to the smallest scales of the velocity field. As the result, he showed that $\Theta \approx \kappa^{-1}$ in the so-called viscous-convective interval, a region where the scales are less than the Kolmogorov length (see Fig. 1). Different experiments dealing with the grid turbulence [5,6] and calculations of the temperature spectrum through numerical simulations (see [7] and references therein) confirm that $\Theta(\kappa)$ follows such these scaling laws.

On the contrary, when Pr is very small, the high fluid conductivity determines quite different situations with respect to the previous ones. Batchelor [4] analyzed the small-scale variations of temperature fluctuations in the case of large conductivity,

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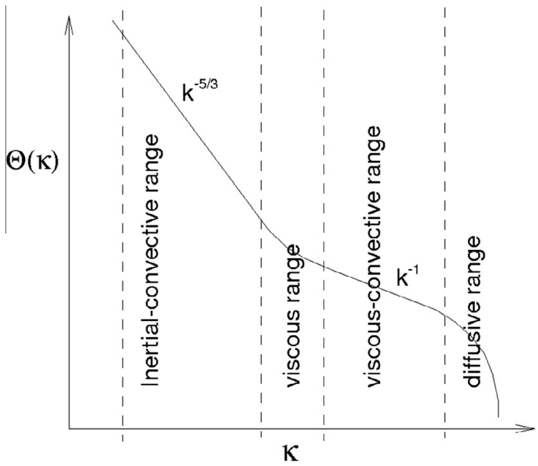

Fig. 1. Scheme of the subranges of the temperature spectrum at high Prandtl numbers.

and found that $\Theta(\kappa) \approx \kappa^{-17/3}$, whereas Rogallo et al. [8] calculated the temperature spectra through numerical simulations of a passive scalar convected by a velocity field with zero correlation time. Rogallo et al. [8] showed that, when the kinetic energy spectrum follows the Kolmogorov law $E(\kappa) \approx \kappa^{-5/3}$, the temperature spectrum varies according to $\Theta(\kappa) \approx \kappa^n$, with $n \approx -11/3$.

According to the experiments of grid turbulence, temperature and velocity correlations are linked with each other when $Pr = O(1)$, whereas the decay rate and characteristic scales depend on the initial conditions. Specifically, Mills et al. [9] obtained very important data about the air turbulence behind a heated grid. They carried out several measurements of nearly isotropic fluctuations of velocity and temperature at different distances from the grid, and recognized that $f_\theta \simeq f$ and $p_* \simeq k$, where f_θ and f are temperature and velocity correlations respectively, p_* is the triple correlation temperature–velocity, and k is the longitudinal triple velocity correlation. Later, Warhaft and Lumley[10] experimentally showed that spectrum shape and decay rate depend upon the initial conditions and that the mechanical–thermal time scale ratio tends to a value close to the unity.

Other important characteristics of $\Theta(\kappa)$ is the self-similarity. This is related to the idea that the combined effect of thermal and kinetic energy cascade in conjunction with conductivity and viscosity, makes the temperature correlation similar in the time. This property was theoretically studied by George (see [11,12] and references therein) which showed that the decaying isotropic turbulence reaches the self-similarity, where $\Theta(\kappa)$ is scaled by the Taylor microscale whose current value depends on the initial condition. Recently, Antonia et al. [13] studied the temperature structure functions in decaying homogeneous isotropic turbulence and found that the standard deviation of the temperature, as well as the turbulent kinetic energy, follows approximately the similarity over a wide interval of length scales. There, the authors used this approximate similarity to calculate the third-order correlations and found satisfactory agreement between measured and calculated functions.

Very important advances, regarding other properties of passive scalars in fully developed turbulence, were recently made [14–19].

Fereday and Haynes [14] studied the decay in a large-scale flow and discussed the relation between the decay obtained by the Lagrangian stretching theories and that calculated with the numerical simulations. Among the other things, the authors determined that the PDF of a passive scalar exhibits algebraic tails, with an exponent of about -3 in a given interval of dimensionless scalar concentration, with a cutoff due to the fluid diffusivity. For what concerns the decay models of a passive scalar, Schekochihin et al. [15] analyzed the case with single-scale random velocity field, and showed that, if there exists separation between flow scale and the box size, the decay rate is the result of the turbulent diffusion of the box-scale. Later, Doering and Thiffeault [16] studied the mixing efficiency of a passive scalar subject to a steady and inhomogeneous source, advected by a statistically homogeneous and isotropic incompressible velocity field. The authors found that the mixing efficiency is limited by the values of $Pr\,R$ and by specific characteristics of the source, and that the scaling laws of the bounds at high $Pr\,R$ depend on the length scales of the source. Tran [17], in an article dealing with the scalar diffusion in shear flows, determined an upper bound for the decay rate of the temperature standard deviation in the case of shear flows with bounded velocity gradients, where the initial temperature distribution is supposed to be a smooth function of the space co-ordinates. Thereafter, Tran [18] analyzed the evolution of the temperature gradient and showed, thanks to the hypothesis of finite velocity gradient, that the square of temperature gradient and its decay rate are both bounded. Next, Burton [19] extended the nonlinear large-eddy simulation method to conditions with moderate and very high Schmidt numbers, and, among the other things, provided the instantaneous field of scalar-energy at viscous-convective scales at high Schmidt-numbers.

From a theoretical point of view, the properties of $\Theta(\kappa)$ can be investigated through its evolution equation. $\Theta(\kappa)$ is the Fourier-Transform of f_θ which varies according to the Corrsin equation [20]. This latter includes G, a term responsible for the

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