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# Directional fields algebraic non-linear solution equations for mobile robot planning

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ABSTRACT

In this study, a novel approach to robot navigation/planning by using half-cell electrochemical potentials is presented. The half-cell electrode's potential is modelled by the Nernst equation to yield automatic search/detection of pipeline flaws by using the direct current voltage gradient (DCVG) technique. We introduce a theory of spherical volumetric electric density in the soil to sustain our postulates for navigational potential fields. The Nernst potential is correlated with the distance to a pipe's flaw by proposing a fitted theoretical-empirical nonlinear regression model. From this, volumetric derivatives are solved as gradient-based fields to control wheeled robot's motion. A nonlinear system for trajectory planning is proposed, and analytically solved by an algebraic solution. This solution directly adjust robot's speed kinematic values to lead it toward the flaw. The inverse/forward kinematic constraints are non-holonomic, and are recursively integrated into the general potential equation. Analytical modelling is reported, and a set of numerical simulations are presented to prove the feasibility of the proposed formulations.


## 1. Introduction

The problem of detecting corrosion for avoiding metals deterioration is a critical issue for industries [1]. In the entire world, there are thousands of kilometres of continental marine pipes to distribute raw natural gas, gasoline, diesel, and so forth [2]. The expenses in resources dedicated to control and prevention of corrosion rise enormously. The DCVG technique [1,3] is an electrochemical measurement technique widely used for detection of defects in the covering of buried pipes. This technique allows the measurement of voltage gradients presented in the soil above a buried pipe. The voltage gradient appears, when a flaw in the covering of the buried pipe, lets a small flow of electric current through the point that is in contact with the soil. The magnitude of the current flow is a function of both, the damaged area and soil conditions. The measurements of voltage gradients are commonly done by a human operator using an analogue voltmeter and two electrodes known as electrodes of half cell or reference electrodes. To measure voltage gradients, it is necessary to apply a DC voltage to the pipe with known characteristics and use it as a signal pattern during the measurements (Fig. 1-left). A drawback of the DCVG technique is that, it is a process carried out by an experienced human operator in the measuring process, who may

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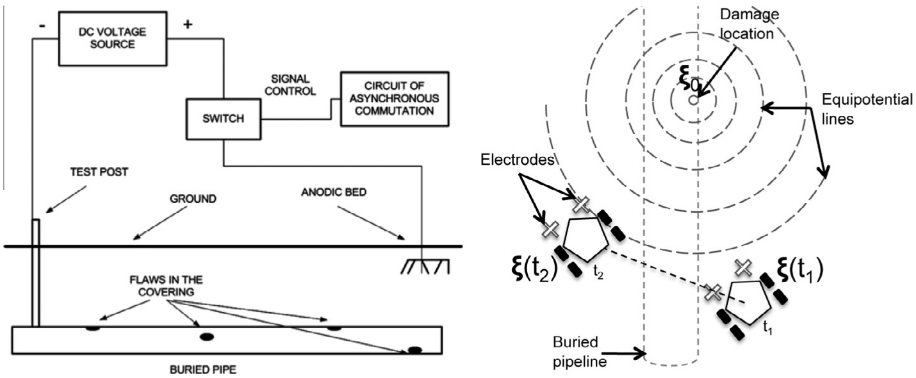


**Fig. 1.** Left: Diagram of DC circuit for measuring buried pipes; Right: Proposed robotised DCVG-based search.

walk hundred of metres attempting to locate flaws. Nevertheless, inspected buried pipelines are as long as dozens or even thousands of kilometres, making this task to some extent intractable for human operators. The research work presented in this article contributes an analysis of a novel planning model and numerical experiments based on the DCVG theory to find an automatic search function for mobile robots. The proposed approach contributes the following summarised issues:

1. A numerical calibration way to yield a theoretical model that fits potential measurements and real distances to the flaw Eqs. (13)–(17).
2. A reformulation of the half-cell Nernst equation (1) as a sensing model to feedback the DCVG search process Eqs. (31)–(33).
3. A vector field equation that arises from an electric field volumetric model (Section 5).
4. The solution of a nonlinear system, Eqs. (34) and (35), in which the exact solution leads the system towards the buried pipe's flaw.
5. A non-holonomic robot's kinematic control law that is integrated to recursively control the robot trajectory (Section 7).

Therefore, the issues to discuss in this manuscript are organised in the following manner. The Section 2 describes very concretely the fundamentals of the DCVG technique, and the reformulation of the general model of the Nernst equation adapted to robots' motion planning models. In Section 3, a calibration model is deduced from an empirical model to fit a theoretical nonlinear approximation. In Section 4, a directional derivative model is deduced to allow the robot moving according to volumetric electric potentials measured in the soil. In Section 5 we describe and deduce an analytical non-linear solution to quickly approach by planning towards the flaw. Section 6 describes the inverse/forward kinematic control law for a non-holonomic structure. Finally, summarised conclusions regarding this manuscript contents are given.

## 2. DCVG as motion planning

The process for detection and location of corroded points in buried pipelines require periodically inserting the electrodes in the soil to obtain the corresponding voltage gradient measurements [3]. The measurements are traditionally made by observation of an analogue voltmeter $\hat{E}_t$. The needle moves indicating a potential difference in the electrodes when a flaw in the covering of the pipe is detected near the electrodes [1]. The asynchronous pattern of the voltage applied to the pipeline is to isolate the measurement readings of some other voltage induced to the pipeline by some near system of cathodic protection (Fig. 1-left). While there is no repetitive-deflection of the voltmeter needle in the same asynchronous pattern, flaws are not present.

The application of the DCVG technique is developed by human operators with the support of data loggers and global positioning systems (GPS) to facilitate the registration and position of the located flaws. According to Fig. 1-right, as same as the procedure developed by humans, we propose the robot to move among different magnitude levels of equipotential lines until reaching the flaw location. The robot is instrumented with a couple of electrodes (half-cell each) to be put in the soil surface at $\xi(t) = (x, y)^T$. The robot moves and gradually approaches $\xi_o = (x_0, y_0)^T$ while measured potential difference approaches zero. Therefore, the Hypothesis 2.1 is sustained on a geometrical spheric equipotential model, in accordance to Definition 4.1. From this statements, the directional field emerge in Corollary 4.1, used as the basis for robot's trajectory planning (Fig. 7-right).

**Hypothesis 2.1.** The DCGV measurement is sustained by a spherical volumetric model that relates distance with potential fields, that may be utilised to exert accelerative attraction motions on a mobile robot to easily meet flaw locations.

Now, we introduce the fundamental concepts of the electrochemistry related to the actual problem, only as far as they are important for understanding at a qualitative level the presented results. We present the novelty of the approach by reformulating the Nernst equation. An analytical solution equation is presented in terms of Cartesian positions $\xi(t)$ w.r.t. electro-