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The role of non-Archimedean epsilon in finding the most efficient unit: With an application of professional tennis players

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ABSTRACT

The determination of a single efficient decision making unit (DMU) as the most efficient unit has been attracted by decision makers in some situations. Some integrated mixed integer linear programming (MILP) and mixed integer nonlinear programming (MINLP) data envelopment analysis (DEA) models have been proposed to find a single efficient unit by the optimal common set of weights. In conventional DEA models, the non-Archimedean infinitesimal epsilon, which forestalls weights from being zero, is useless if one utilizes the well-known two-phase method. Nevertheless, this approach is inapplicable to integrated DEA models. Unfortunately, in some proposed integrated DEA models, the epsilon is neither considered nor determined. More importantly, based on this lack some approaches have been developed which will raise this drawback.

In this paper, first of all some drawbacks of these models are discussed. Indeed, it is shown that, if the non-Archimedean epsilon is ignored, then these models can neither find the most efficient unit nor rank the extreme efficient units. Next, we formulate some new models to capture these drawbacks and hence attain assurance regions. Finally, a real data set of 53 professional tennis players is applied to illustrate the applicability of the suggested models.

1. Introduction

In recent years, due to the complexity of issues, the high volume of data, the effects of external factors on the performance of manufacturing or service units, as well as the intense competition in global business environment, any individual decision will encounter enormous challenges if scientific methods are not applied. Considering the relationship between performance and affecting factors, a function called production function has been introduced which leads to the possible discrimination of efficient or inefficient units. The production function provides maximum outputs for all combinations of inputs and can be estimated by two main methods: parametric and nonparametric. The nonparametric approaches versus the parametric approaches have received a considerable amount of interest, both theoretically and practically. In parametric approaches, a specific functional form of the production function must be considered, and accordingly the parameters of this form have to be estimated. Notwithstanding this, in nonparametric approaches these types of assumptions are relaxed. Indeed, the name of 'nonparametric' refers to the non-existence of such parameters in production function (for more details see [1]).

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Charnes et al. [2] introduced data envelopment analysis (DEA), as a nonparametric technique for approximating the production function, to assess the relative efficiency of homogeneous groups of operating decision making units (DMUs). Banker et al. [3] extended this approach from constant returns to scale (CRS) to variable returns to scale (VRS) assumption.

Charnes et al. [4] explained that if inputs and outputs multipliers drop to zero, then the corresponding inputs and/or outputs would be excluded from the efficiency score computations. In this situation, the number of efficient DMUs becomes unreasonably large and the discriminating power of DEA would be reduced. To capture this drawback, the non-Archimedean infinitesimal epsilon was introduced. As it has been mentioned in Charnes et al. [5]:

> "... *if one uses a small number in place of the infinitesimal epsilon, one is caught between Scylla and Charybdis, i.e. for decent convergence to an optimum, the numerical zero tolerance should be as large as possible, whereas the numerical value approximating the infinitesimal should be as small as possible!*"

it is so important to determine a suitable value for epsilon. Ali and Seiford [6] proposed an upper bound on epsilon; however, Mehrabian et al. [7] showed that this upper bound is invalid and presented a procedure to determine an assurance interval for the epsilon. Solving only one linear programming (LP) problems was needed to determine the assurance interval. Amin and Toloo [8] introduced a polynomial-time algorithm for computing an assurance value for epsilon without needing to solve an LP model.

In conventional DEA models usually more than one efficient DMU are recognized, and these models fail to provide more information about the efficient DMUs. In order to receive such information, numerous ranking methods have been proposed. Adler and Friedman [9] comprehensively reviewed all ranking methods in DEA context and divided them into six various groups. On the other hand, to find the most efficient unit it is not necessary to determine all efficient units, rank them, and then select a DMU with the greatest ranking score. Hence, an integrated DEA model can be utilized to obtain the optimal common set of weights. These weights help us to measure the most efficient unit in an identical condition. A number of authors have conducted various studies on the problem of finding an optimal common set of weights in the DEA literature: Li and Reeves [10], Karsak and Ahiska [11], Ertay et al. [12], Amin and Toloo [13], Toloo and Nalchigar [14] and Toloo [15], Toloo [16]. To determine the most efficient DMU the proposed integrated mixed integer linear programming (MILP) model of Amin and Toloo [13] can be applied, instead of solving n LPs. This model enables us to get rid of a parameter in the objective function which is used by Ertay et al. [12]. Afterwards, Amin [17] explained a drawback of the MILP model of Amin and Toloo [13] and proposed a new mixed integer nonlinear programming (MINLP) model to overcome it. Nonetheless, the applicability of this model was illustrated by a real data set of the facility layout design (FLD) problem taken from Ertay et al. [12]. This model includes the non-Archimedean epsilon, however Amin [17] neglected to mention how an assurance value should be calculated. Foroughi [18] claimed that in some cases, the model of Amin [17] is infeasible and to capture this drawback formulated a new MILP model. He also proved the proposed model is always feasible and utilized this model to rank extremely efficient DMUs, as well.

Toloo [15] improved Amin's MINLP model [17] by introducing an MILP model and also extended it from CRS to VRS. Toloo [16] formulated an MILP model for finding the most efficient DMU when there is no evident input. Recently, Toloo and Ertay [19] suggested a new approach for finding the most cost efficient DMU and applied it on automotive industry in Turkey.

There is an important issue should be mentioned here. The well-known two-phase method is proposed to get rid of specifying a value for the non-Archimedean epsilon in conventional DEA model (for full details see [20]). Nevertheless, this approach is useless for integrated DEA models. It places emphasis on the role of non-Archimedean epsilon in these types of DEA models.

The rest of the present paper is organized as follows: the literature is reviewed in Section 2. Section 3 introduces a new MINLP model to determine the non-Archimedean epsilon for Amin's model [17]. In Section 4, we illustrate some drawbacks of Foroughi's model [18] and propose a new MILP model to overcome it. The penultimate section applies the developed approaches to a real data set involving 53 professional tennis players. Conclusions and remarks are provided in the last section.

2. Literature review

Consider a set of n peer DMUs which each DMU_j $(j = 1, \ldots, n)$ consuming varying amounts of m inputs x_{ij} $(i = 1, \ldots, m)$ in the production of s outputs y_{rj} $(r = 1, \ldots, s)$. The multiplier form of CCR model for under evaluation unit DMU_o is as follows:

$$
\begin{aligned}
\max \quad & \sum_{r=1}^{s} u_r y_{ro} \\
\text{s.t.} \quad & \\
& \sum_{i=1}^{m} v_i x_{io} = 1 \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0, \quad j = 1, 2, \ldots, n \\
& v_i \geqslant \varepsilon^*, \quad i = 1, 2, \ldots, m \\
& u_r \geqslant \varepsilon^*, \quad r = 1, 2, \ldots, s
\end{aligned}
\tag{1}
$$