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Investigation of the fractional diffusion equation based on generalized integral quadrature technique

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ABSTRACT

Nowadays, the conventional Euclidean models are mostly used to describe the behavior of fluid flow through porous media. These models assume the homogeneity of the reservoir, and in naturally fractured reservoir, the fractures are distributed uniformly and use the interconnected fractures assumption. However, several cases such as core, log, outcrop data, production behavior of reservoirs, and the dynamic behavior of reservoirs indicate that the reservoirs have a different behavior other than these assumptions in most cases. According to the fractal theory and the concept of fractional derivative, a generalized diffusion equation is presented to analyze the transport in fractal reservoirs. Three outer boundary conditions are investigated. Using exact analytical or semi-analytical solutions for generalized diffusion equation with fractional order differential equation and a fractal physical form, under the usual assumptions, requires large amounts of computation time and may produce inaccurate and fake results for some combinations of parameters. Because of fractionality, fractal shape, and therefore the existence of infinite series, large computation times occur, which is sometimes slowly convergent. This paper provides a computationally efficient and accurate method via differential quadrature (DQ) and generalized integral quadrature (GIQ) analyses of diffusion equation to overcome these difficulties. The presented method would overcome the imperfections in boundary conditions' implementations of second-order partial differential equation (PDE) encountered in such problems.

1. Introduction

The 60% of the remaining oil in the world is found in carbonate reservoirs. Carbonate reservoirs specifically include heterogeneous natural fractures on a wide range of spatial scales. These types of reservoirs that are heterogeneous cannot be easily described and the paths of flow are unpredictable. In spite of these complexities, the pressure transient models are developed under the assumption of standard geometry and homogeneity behavior, which are not realistic in most cases

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Nomenclature

$A_{ik}^{(m)}$	weighing coefficient of the mth-order derivative of p_D with respect to r_D
$\bar{A}_{jk}^{(n)}$	weighing coefficient of the nth-order derivative of p_D with respect to t_D
B	oil formation volume factor, RB/STB [L^3/L^3]
c_k^{ij}	weighing coefficient of the integral of p_D with respect to t_D
c_t	total isothermal compressibility factor, psi^{-1} [Lt^2/m]
d	fractal topological dimension
h	net formation thickness, ft [L]
k	reservoir rock permeability, md [L^2]
M	number of grids points in t_D direction
N	number of grids points in r_D direction
p_i	initial reservoir pressure, psi [$m/(Lt^2)$]
p_D	dimensionless pressure
p_{wf}	wellbore flowing pressure, psi [$m/(Lt^2)$]
q	flow rate, STB/D [L^3/t]
r	distance from the center of wellbore, ft [L]
r_D	dimensionless radius
r_{Di}	ith dimensionless grid point in r_D direction
r_{eD}	dimensionless external radius
r_w	wellbore radius, ft [L]
t_D	dimensionless time
t_{Di}	ith dimensionless grid point in t_D direction
v_r	radial velocity, ft/s [L/t]
θ	fractal dynamical index
μ	oil viscosity, cp [m/(Lt)]
ρ	oil density, lb_m/ft^3 (m/L^3)
ϕ	porosity of reservoir rock

[1–9]. One of the aspects of the geometrical complexity is that flow distribution is affected by the distribution of fractures most of the times. There may be some regions in the reservoir with a group of fractures whereas the reservoir does not contain fractures. The different scale of fractures shows an uncertainty relevant element in developing the mathematical model of reservoir. Thus, according to these high complexities, the weakness of the Euclidean models is clear in most cases. Alternatively, fractal geometry provides an appropriate approach to explain and model the complex fractured reservoirs [9,10].

Chang and Yortsos [11] were the first scholars who presented a mathematical model to describe the pressure behavior of transport in fractally fractured reservoirs. They also obtained the analytical results, which showed that a fractal reservoir can be identified by a log–log straight line with the slope equal to a function of the fractal topological dimension and the fractal dynamical index. Beier [12] and Aprilian [13] applied fractal reservoir model to analyze well test data for complex reservoirs which could not be matched by traditional model, and the results were consistent with field practice. Poon [14] extended the concept of fractal distribution to find out the effect of a composite reservoir. He et al. [15] established a fractal model for unsteady-state flow in double-porosity and permeability reservoirs based on Warren and Root [16] model, and solved it by the Correction Prediction method. Meanwhile, they analyzed pressure performances and their effect on different factors. On the basis of Warren and Root model, Zhang et al. [17], set up a model for deformed double-porosity fractal gas reservoirs by introducing fractal parameters and compressibility factors. They solved this model by finite element method with consideration of secondary boundary conditions, plotting the results into type curves. To consider threshold pressure gradient in low permeability reservoir and gradual pressure propagation in the formation, Hou and Tong [18] established the non-Darcy flow model for deformed double-porosity media [19]. The production of oil from fractally fractured systems indicates that the system has an anomalous behavior, which cannot be modeled by the classical diffusion equation. Since the history of flow is pivotal in all stages of production, the fractional derivative can be used as an appropriate approach to consider the history of flow in the mathematical model [20].

Since most analytical, semi-analytical, and numerical methods are slowly convergent and may produce inaccurate results to differential equations, Bellman et al. [21] introduced another numerical method that gives acceptable accurate results using small number of grid points [22–26] This alternative method was called differential quadrature method (DQM) and was further developed by Bellman and Roth [27]. The double-porosity reservoirs were discussed in Ref [9], and the response of these reservoirs assuming fractal structure was analyzed. A model was developed for analysis of double-porosity reservoirs where it was passed up the effect of fractal topological dimension (d), which was assumed to be equal to Euclidean dimension (D). The mathematical model was analyzed by using the Laplace transform.

This paper establishes the solution of generalized diffusion equation for single-porosity naturally fractured reservoirs based on a new application of DQM and generalized integral quadrature (GIQ). The method weighting coefficients are not exclusive and any methods that can be used in conventional DQM for evaluation of the weighting coefficients, such as