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Efficient modal dynamic analysis of flexible beam–fluid systems

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A B S T R A C T

This paper proposes an efficient simplified method to determine the modal dynamic and earthquake response of coupled flexible beam–fluid systems and to evaluate their natural vibration frequencies. The methodology developed extends available analytical solutions for mode shapes and natural vibration frequencies of slender beams with various boundary conditions to include the effects of fluid–structure interaction. The proposed method is developed for various beam boundary conditions considering lateral interaction with one or two semi-infinite fluid domains. Numerical examples are provided to illustrate the application of the proposed method, and the obtained results confirm the importance of accounting for fluid–structure interaction effects. We show that the developed procedure yields excellent results when compared to more advanced coupled fluid–structure finite element solutions, independently of the number of included modes, beam boundary conditions, and number of interacting fluid domains. The proposed simplified method can be easily implemented in day-to-day engineering practice, as it constitutes an efficient alternative solution considering the fluid–structure modeling complexities and related high expertise generally involved when using advanced finite elements.

1. Introduction

Several civil engineering and industrial applications involve the vibrations of beam-like structures in contact with water or fluid domains, including dams, navigation locks, quay walls, break-waters, offshore platforms, drilling risers, liquid storages, nuclear reactors, oil refineries, petrochemical plants, fuel storage racks, etc. This popular topic has attracted many researchers over the last decades and various approaches were proposed, varying from simplified to more complex analytical and numerical formulations. Neglecting structural flexibility, Westergaard [1] introduced the added-mass concept and applied it to a dam-reservoir system, and later Jacobsen [2] and Rao [3] generalized the added-mass formulation to evaluate hydrodynamic effects on rigid circular and rectangular piers vibrating in water. Goto and Toki [4] and Kotsubo [5] evaluated the hydrodynamic pressures induced by harmonic motions on circular or elliptical cross-section cylindrical towers along rigid body and deformed mode shapes. Chopra [6,7] showed that a dam's flexibility influences significantly its interaction with the impounded reservoir, and consequently the overall dynamic and seismic responses. Other studies also confirmed the importance of accounting for structural flexibility and fluid–structure interaction [8–11]. Han and Xu [12] developed an analytical model to compute the modal properties of a slender flexible cylinder vibrating in water, and proposed a simplified added-mass formula to compute its natural frequencies. Other researchers studied the sensitivity of the hydrodynamic response of cantilever structures to various factors, such as: (i) a tip mass or inertia concentrated at one end [13–16],

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(ii) a restrained boundary condition at the base of the beam [18,17,16], and (iii) non-uniformity of beam cross-section [19,17,16].

Most of the previous studies focused on cantilever structures surrounded by fluid. Work on beam-like structures interacting with 2D semi-infinite fluid domains was mainly related to dam monoliths impounding water reservoirs, while fewer researchers investigated the behavior of slender beams subjected to hydrodynamic loading of this type. Xing et al. [20], Zhao et al. [21] and Xing [22] developed analytical formulations to examine the dynamic response of a cantilever flexible beam interacting with a 2D semi-infinite water domain, and discussed the effects of various boundary conditions of the fluid domain. Nasserzare et al. [23] proposed a procedure to extract the natural frequencies and modes of a dry structure from vibrational data containing fluid–structure interaction effects, and they applied the methodology to a beam–water system. De Souza and Pedroso [24] developed a finite element procedure to determine the coupled dynamic response of a Bernoulli beam interacting with a 2D acoustical cavity, and they validated the vibration frequencies and modes obtained by comparing to other finite element and analytical solutions.

The present paper is motivated by the need to develop simplified methods extending results from classical vibration beam theory to include the effects of 2D hydrodynamic forces on one or both sides of a vibrating beam. The majority of the previous work and other relevant literature addressed hydrodynamic effects on cantilever beams, fully clamped or partially restrained at the base, and little attention has been given to other boundary conditions such as pinned or sliding supports. Most of the previous studies also focused on the determination of the modal properties of a vibrating beam interacting with a fluid, while less concern has been devoted to the time evolution of beam's earthquake response indexes indicators such as displacements, shear forces, and bending moments. These restrictions will be addressed in this paper.

2. Modal dynamic response of a beam–fluid system

2.1. Basic assumptions and notation

Fig. 1 shows a slender beam of height H vibrating in contact with fluid on one or both sides. We adopt a Cartesian coordinate system with origin at the base of the beam, a horizontal axis x and a vertical axis y coincident with the axis of symmetry of the beam. The semi-infinite fluid domains have a rectangular geometry with constant height equal to that of the beam. We denote by Λ_f the number of fluid domains in contact with the beam, and by left and right side fluid domains those extending from the beam towards negative and positive x directions, respectively. Both configurations illustrated in Fig. 1(a) and (b) will be investigated here, i.e. $\Lambda_f = 1$ and $\Lambda_f = 2$, respectively. The beam will be referred to as wet when it is in contact with one fluid domain at least, i.e. $\Lambda_f > 0$, and dry otherwise. The boundary conditions of the beam can be Clamped–Free (CF) as shown in Fig. 1 or Clamped–Pinned (CP), Clamped–Sliding (CS), Clamped–Clamped (CC) or Pinned–Pinned (PP) as illustrated in Fig. 2. Points at the top, middle and base of the beam are denoted by A, B and C, respectively, as shown in Fig. 1(a). These points will be used later to illustrate various dynamic responses of the studied beams. We also assume that: (i) the beam is slender so that only flexural deformations are considered, i.e. Euler–Benoulli beam theory is used; (ii) the beam is made of a linear, homogenous, and isotropic elastic material; (iii) only small deflections normal to the undeformed beam axis are included; (iv) the fluid is incompressible and inviscid, with its motion irrotational and limited to small amplitudes, (v) the fluid remains in full contact with the beam during vibration due to the assumption of small displacements, and (vi) gravity surface waves and non-convective effects are neglected.

2.2. Governing equations

We first assume that the beam–fluid system is subjected to a unit harmonic free-field horizontal ground motion $\ddot{u}_g(t) = e^{i\omega t}$, where ω denotes a forcing frequency, t the time variable and i the unit imaginary number. Using a modal superposition analysis, the frequency response functions along beam's height of lateral displacement u, lateral acceleration $\ddot{u}$, bending moment $\mathcal{M}$ and shear force $\mathcal{V}$ can be expressed as

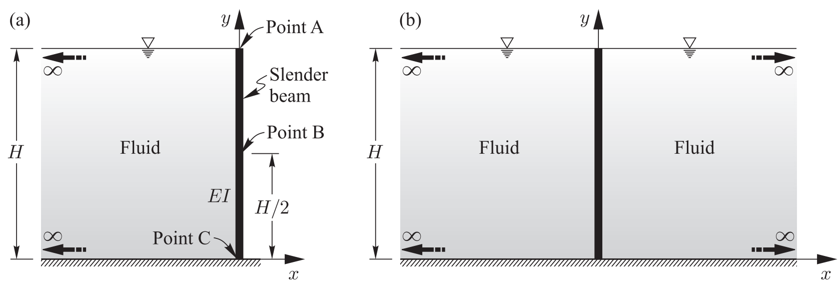

Fig. 1. Slender beam vibrating in contact with a fluid acting on: (a) one side and (b) both sides.