Applied Mathematical Modelling 39 (2015) 128–136

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Integral transform solutions to the bending problems of moderately thick rectangular plates with all edges free resting on elastic foundations

Bin Tian [a,*], Rui Li [b,*], Yang Zhong [c]

[a] Department of Science and Technology, China Road and Bridge Corporation, Beijing 100011, PR China
[b] State Key Laboratory of Structural Analysis for Industrial Equipment, Department of Engineering Mechanics, Dalian University of Technology, Dalian 116024, PR China
[c] Faculty of Infrastructure Engineering, Dalian University of Technology, Dalian 116024, PR China

ARTICLE INFO

Article history:
Received 12 November 2012
Received in revised form 19 January 2014
Accepted 16 May 2014
Available online 27 May 2014

Keywords:
Moderately thick plate
Finite integral transform
Analytical solution

ABSTRACT

Double finite integral transform method is proposed in this paper to obtain the analytical bending solutions of moderately thick rectangular plates with all edges free resting on elastic foundations, which was difficult to be acquired due to the complexity of the mathematical model. The accuracy of the method is validated by the literature as well as the finite element method (FEM). The solution approach presented here can be used to solve the problems of plates with different boundary conditions in an elegant, step-by-step way without predetermining any trial solutions. The results obtained in this paper can serve as benchmarks for future reference.

© 2014 Elsevier Inc. All rights reserved.

1. Introduction

The most commonly used moderately thick plate theories are those proposed by Reissner [1,2] and Mindlin [3], both of which incorporate the effect of transverse shear deformation. Based on these theories, extensive researches on seeking the bending solutions of rectangular thick plates have been conducted. Although the endeavor has been made for many years, only few analytical solutions were obtained such as for the plates with the opposite edges simply supported [4], which is due to the complexity of the governing equations as well as the boundary conditions. For the plates with the other combinations of boundary conditions, the analytical solutions are still not desirable.

The aim of this paper is to present the benchmark analytical solutions of the rectangular thick plates with all edges free resting on elastic foundations. There has been a large literature on solving the bending problems of such plates with numerical methods [5–10]. Henwood et al. [5] developed a finite difference bending solution of a thick rectangular plate resting on a Winkler foundation and carrying an arbitrary transverse load using a central difference scheme. Qin [6] established a hybrid Trefftz finite element model to obtain the numerical solutions for thick plates on elastic foundation based on a modified variational principle. Rashed et al. [7] applied the boundary element method to thick plate resting on Winkler foundations, where the fundamental solutions were constructed using operator decoupling technique. Liu [8] performed the static analysis of rectangular thick plates on Winkler foundation by developing the two-dimensional differential quadrature element method, by which some numerical results for SSSS, CCCC and SFSF plates were presented. Buczkowski and

* Corresponding authors. Tel.: +86 15840912019.
E-mail addresses: tianbin_0712@163.com (B. Tian), ruili@dlut.edu.cn (R. Li).

http://dx.doi.org/10.1016/j.apm.2014.05.012
0307-904X/© 2014 Elsevier Inc. All rights reserved.

Torbacki [9] built a finite element model for bending analysis of rectangular and circular thick plates resting on a two-parameter elastic foundation. Ferreira et al. [10] dealt with the bending of simply supported or clamped thick plates on Winkler foundations by a wavelet collocation method which is based on the use of the autocorrelation functions of Daubechie's compactly supported wavelets. Compared to various numerical solutions, there are few references dealing with the problems by analytical methods. Henwood et al. [11] derived the Fourier series solution for the bending of thick rectangular plates resting on Winkler foundations by the sum of two trial double series and a particular solution. Shi et al. [12] analyze the similar plate on a Pasternak foundation using the superposition method. Of so many achievements on the related problems, we have named a few but this never obscures the importance of the others.

Integral transform technique is one of the effective tools to solve the partial differential equations [13], which has been applied successfully to obtain the analytical solutions of some engineering problems, including those based on the theory of elasticity [14]. In recent years, the finite integral transform method has been extended for bending and vibration analysis of rectangular thin plates [15–17], which demonstrated the rationality and accuracy compared to the conventional analytical methods such as the semi-inverse method. Consequently, more analytical solutions which were unavailable are expected to be obtained. It is the purpose of this paper to derive the double finite integral transform solutions to the title problems, which have not been reported before, to the knowledge of the authors. Several examples are given to verify the method and to present the benchmark results for future comparison. The method is not limited to specific issues like the title problems, but applicable to the plates with any other combinations of boundary conditions. Further studies on more unsolved boundary value problems are being conducted with the present approach.

2. Analytical bending solutions of moderately thick rectangular plates with all edges free resting on elastic foundations

2.1. Governing equations and the boundary conditions

Fig. 1 shows the coordinate system of a free moderately thick rectangular plate, with the dimensions a and b in the x and y directions while h in the z direction running downward from plate mid-plane, resting on an Winkler foundation with the foundation modulus K. Based on the Mindlin plate theory [3], the governing equations for the bending of such a plate are

$$\begin{aligned}
&\frac{\partial^2 W}{\partial x^2}+\frac{\partial^2 W}{\partial y^2}-\frac{\partial \varphi_x}{\partial x}-\frac{\partial \varphi_y}{\partial y}+\frac{K}{C}W+\frac{q}{C}=0,\\
&\frac{\partial^2 \varphi_x}{\partial x^2}+\frac{1-\mu}{2}\frac{\partial^2 \varphi_x}{\partial y^2}+\frac{1+\mu}{2}\frac{\partial^2 \varphi_y}{\partial x\partial y}+\frac{C}{D}\left(\frac{\partial W}{\partial x}-\varphi_x\right)=0,\\
&\frac{\partial^2 \varphi_y}{\partial y^2}+\frac{1-\mu}{2}\frac{\partial^2 \varphi_y}{\partial x^2}+\frac{1+\mu}{2}\frac{\partial^2 \varphi_x}{\partial x\partial y}+\frac{C}{D}\left(\frac{\partial W}{\partial y}-\varphi_y\right)=0,
\end{aligned} \qquad (1a-c)$$

where W, φ_x and φ_y are the transverse deflection of the plate, rotations of the normals to the mid-surface about the y and x axes, respectively; C and D are the shear stiffness and flexural rigidity; μ is the Poisson's ratio; and q is the intensity of distributed transverse load.

The boundary conditions at the edges $x=0$ and $x=a$, where the internal forces are represented by the generalized displacements, are

$$\begin{aligned}
&M_x|_{x=0,a}=-D\left(\frac{\partial \varphi_x}{\partial x}+\mu\frac{\partial \varphi_y}{\partial y}\right)=0,\\
&M_{xy}|_{x=0,a}=-\frac{D}{2}(1-\mu)\left(\frac{\partial \varphi_x}{\partial y}+\frac{\partial \varphi_y}{\partial x}\right)=0,\\
&Q_x|_{x=0,a}=C\left(\frac{\partial W}{\partial x}-\varphi_x\right)=0,
\end{aligned} \qquad (2a-c)$$

while at $y=0$ and $y=b$,

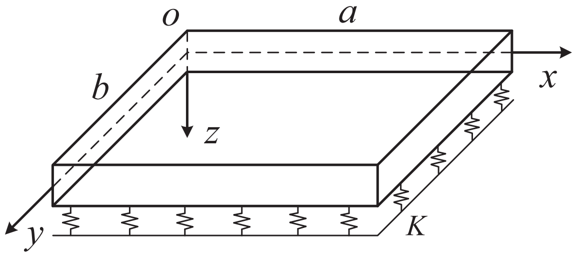

Fig. 1. A free moderately thick rectangular plate resting on an elastic foundation.

Download English Version:

https://daneshyari.com/en/article/1703688

Download Persian Version:

https://daneshyari.com/article/1703688

Daneshyari.com