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Optimal ordering quantities for substitutable products with stock-dependent demand

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ABSTRACT

Many retailers are trying to increase their product offers to compete for market share. However, the offer of similar products implies that these products may be substitutable to the consumer. In this paper, we study an inventory control problem in which demand is satisfied by using two mutually substitutable products. Since the products are substitutable, in case of a stock-out for one of them, a known fraction of its demand can be satisfied by using the stock of the other product. The demand for each product also depends on the inventory levels of both of the products at a certain time. The orders for both products are placed jointly. Our aim is the determination of the order quantity for each product that maximizes the joint profit function. Some numerical results, which have several interesting managerial insights and implications, are presented and discussed.

1. Introduction

The occurrence of temporary stock-outs at retail is common in frequently purchased product categories. For example, a recent study in U.S.A supermarkets found that 8.2% of items on average were unavailable to the consumer on a typical afternoon [1]. It is common in such cases that consumers, who are seeking to buy a certain product, will be willing to substitute with a similar product when facing a stock-out, rather than visiting a different store to find the original product. A survey reports that only 12–18% of shoppers said that they would not buy an item on a shopping trip if their favorite brand-size was not available; the rest indicated that they would be willing to buy another size of the same brand, or switch brands [1]. Hence, it is in a retailer's interest to take into account this substitution effect when making inventory decisions. It is well known that efficient management of inventories across the different facilities in a supply chain is critical to increase profits [2,3]. So, ordering rules play an important role in controlling inventory level and determining order quantities.

There is a vast amount of literature in the area of product substitution. This literature has been classified into many categories. We will mention some of these major, in our opinion, streams in order to try to categorize our work. One major classification has been made depending on whether the substitution is driven by the consumer (consumer driven substitution) or by the decision-maker (supplier driven substitution, also called the transshipment problem).

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Another classification is based on the reason for which the substitution is made. According to Tang and Yin [4] there are three types of product substitution: stock-out based substitution, assortment-based substitution, and price-based substitution. Stock-out based substitution corresponds to a situation in which a customer may purchase another product as a substitute, when the preferred product is out of stock. Assortment-based substitution occurs when products with similar attributes are substitutable. Price-based substitution corresponds to a situation in which a retailer uses differential pricing to make certain products substitutable.

Another well-recognized categorization of this literature is based on whether the authors consider one- or two-way substitution [5]. One-way substitution assumes that products can be ordered based on an attribute such as quality, or speed of service. So, products with higher levels of the attribute can substitute for products with lower level of the attribute. This is usually applied on supplier driven substitution problems. On the other hand, two-way substitutability enables consumers to substitute among products within the same category.

Taking into account the previously mentioned categories, our study relates to the area of *stock-out based, customer driven, two-way substitution.* Notable papers in this area are those of McGillivray and Silver [6], who are one of the first to study this phenomenon by assuming items with identical cost and a fixed substitution probability. They evaluate the optimal order quantity by simulation and heuristics. Parlar and Goyal [7] study the same problem and show that the total expected profit is concave for a wide range of problems. Moreover, Parlar [8] proves the existence of Nash equilibrium for the case of two substitutable products with stochastic demands. Ernst and Kouvelis [9] study demand substitution in stockout situations between individual products and packaged goods. Drezner et al. [10] present an EOQ model comparing the cases of full substitution and no substitution. Rajaram and Tang [11] develop a heuristic and examine how the level of demand variation and correlation, and the degree of substitution affect the order quantities and expected profits under substitution. Netessine and Rudi [12] consider an arbitrary number of products under both centralized inventory management and competition. Substitution is modeled by letting the unsatisfied demand for a product flow to other products in deterministic proportions. They analytically confirm the numerical results of Rajaram and Tang [11] and Ernst and Kouvelis [9]. More recently, Nagarajan and Rajagopalan [13] derive the optimal inventory policy in a model with two substitutable products whose demands are negatively correlated in both single-period and multi-period scenarios. Tang and Yin [4] develop a model with deterministic demand. They examine how a retailer should jointly determine the order quantity and the retail price of two substitutable products under fixed and variable pricing strategies. Hsieh and Wu [14] establish a model for a supply chain, which consists of two suppliers with capacity uncertainties. The suppliers sell substitutable products through a common retailer who faces random demand for these two products. Maity and Maiti [15] study an inventory control model with stock dependent demand for deteriorating multi-items which are either complementary and/or substitutes. Gurler and Yilmaz [16] consider a two level supply chain model with a newsboy setting for two substitutable products. They assume that the retailer is allowed to return some or all of the unsold products to the manufacturer with some credit.

Another important factor that a retailer should take into account is the demand stimulation effect. It has been observed in supermarkets that demand is usually influenced by the amount of stock displayed on shelves. Balakrishnan et al. [17] suggest that high inventories might stimulate demand for a variety of reasons such as "increasing product visibility, kindling latent demand, signaling a popular product, or providing consumers an assurance of future availability". Inventory models with stock dependent demand have concerned the interest of researchers in the recent years. Following the basic version of the models of this type by Gupta and Vrat [18], a large number of mathematical models were proposed by a range of researchers including Giri et al. [19], Datta and Paul [20], Zhou and Yang [21], Dye et al. [22], Hsieh and Dye [23], Dye and Hsieh [24]. Readers may refer to Urban [25] for a comprehensive review of inventory models with stock dependent demand.

In the case of substitutable products, it is logical to assume that the demand for each product will not only be affected by its own inventory level but also by the inventory level of the other similar products. When facing such a demand, a retailer must way the benefits of increased revenue from higher inventories against higher inventory costs. Recently Stavrulaki [5] studied the joint effect of demand stimulation and product substitution on inventory decisions by considering a single-period, stochastic demand setting.

Many firms, like supermarkets, order a group of items simultaneously, rather than individually. By coordinating the replenishment of several items, cost savings can be obtained by sharing the setup cost between the items. In many practical circumstances, it is preferable to jointly order and replenish items because they may share the same supplier or use the same means of transport. In addition, a synchronized inventory cycle could be appropriate in a vendor-managed inventory setting where vendor supplies similar products.

In this paper we study an inventory control problem in which a retailer satisfies demand using two similar products. These products are ordered jointly by the same supplier in each replenishment cycle. We assume the products are mutually substitutable. So in case of a stock-out for one of the products, a known fraction of its demand can be satisfied by using the stock of the other product. The demand for each product depends on the inventory levels of both of the products at a certain time and the products inventories deplete simultaneously. The retailer must decide on the order quantity of each product in order to maximize his profit. Our basic model assumes that all the cost and profit parameters are different for the two products. This happens for a set of products with same functions and quality levels but various brands, such as food products in supermarkets shelves. Also our model is valid for the symmetric cost and profit case which is appropriate for horizontally differentiated products, such as various ice cream or yogurt flavours. In addition, as a special case, we study the model with