Applied Mathematical Modelling 39 (2015) 230–243

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Modeling and forecasting river flows by means of filtered Poisson processes

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ARTICLE INFO

Article history:
Received 19 February 2013
Received in revised form 3 January 2014
Accepted 20 May 2014
Available online 2 June 2014

Keywords:
Response function
Estimation
Forecasting
Conditional expectation
Autoregressive model

ABSTRACT

To model and forecast daily river flows, filtered Poisson processes with a response function that generalizes the one commonly used in hydrology are considered. The form of the response function is based on the concept of the instantaneous unit hydrograph and improves the quality of the classical model. A statistical procedure based on the theoretical autocorrelation coefficients is used to estimate the model parameters. Then, the quality of the model is assessed by comparing the theoretical autocorrelation coefficients obtained with both the generalized and the classical models to the corresponding empirical coefficients. Furthermore, an estimator of the flow at time $t+1$, given the flows at time $t-1$ and t, is developed in a particular case and the forecasting power of the model is checked, using some indicators of difference, compared to the classical model and to an autoregressive model. An application to the Delaware and the Hudson Rivers, located in the United States, is presented.

1. Introduction

Mathematical modeling of river flows is intended to represent the corresponding hydrological subsystem as a function that takes into account the physical aspect of the system connecting the input variables to the output variables. This representation is based primarily on stochastic models that describe, albeit simply, the dynamic of a streamflow. The formulation of a stochastic model to simulate river flows at short time intervals (say, daily) is the subject of this paper.

The literature on stochastic modeling for simulation of daily river flows, in particular, is extensive. Important papers on this subject include Nash [1], who formalized the linear theory of the instantaneous unit hydrograph (denoted by IUH), and Bernier [2] and Weiss [3,4], who proposed to use filtered Poisson processes. These studies are based on the notion of the IUH, under the assumption that the only input variable in the system is precipitation.

The concept of a linear reservoir storage is the method upon which Nash [1] based his work to propose a model of the IUH. It assumes that the trajectory of the flow in a watershed after a rainfall is equivalent to the instantaneous flow through a succession of n linear reservoirs; the output from a reservoir becomes the input for the next. The IUH corresponding to the Nash [1] model can be obtained by the gamma distribution function with two parameters, $\Gamma(n, K)$, where K is the storage coefficient. The outflow, discontinuous for the first reservoir, becomes gradually continuous as it travels through the n reservoirs.

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http://dx.doi.org/10.1016/j.apm.2014.05.027
0307-904X/

Bernier [2] was the first to propose the filtered Poisson process, also called "shot noise", as a model. He considered the intermittent process pattern and the Poisson process of effective rainfall events, together with a deterministic function relating rainfall and daily discharge. The flow can evolve deterministically when there is no more incentive (precipitation) on the watershed and follows a trajectory obeying a particular law; it is the dry period. Precipitation is considered as the important element within the random process of the flow formation. Periods of flood recession flows follow alternatively periods of growth flows caused by the supply basin due to precipitation.

Weiss [3,4] studied in more detail the "shot noise" or filtered Poisson process by developing it as a simple and physically realistic model that reproduces recession daily flows. The model developed is equivalent to a first order autoregressive process, but in which innovations are not Gaussian. The structure of this model is not valid for complex hydrological regimes, where other random mechanisms combine to rain, like snow melt and temperature. However, the shape of the hydrograph can be realistic for flow recession periods.

Bernier [2] carried out a review of the models used to simulate daily river flows; see also Bodo and Unny [5]. According to Lawrance and Kottegoda [6], the generation of synthetic daily flows concerns the modeling of a series of dependent and non-Gaussian variables with a high variability. In fact, the daily flows are characterized by the presence of intermittent patterns of rain events and by the skewed nature of the hydrograph, and then by rapid increases followed by slow recessions. The model proposed by Lefebvre and Guilbault [7] aims at reproducing the movement of ascension–recession of the hydrograph through a stochastic conceptual model.

Now, let $\{N(t), t \geqslant 0\}$ be a Poisson process with rate λ. A filtered Poisson process is a stochastic process $\{X(t), t \geqslant 0\}$ defined by (see Parzen [8], for instance)

$$X(t) = \sum_{n=1}^{N(t)} w(t, \tau_n, Y_n) \quad (X(t) = 0 \text{ if } N(t) = 0), \tag{1}$$

where w is called the response function; in our application, it gives the value at time t of an event of magnitude Y_n of the Poisson process that occurred at time τ_n. A response function that is commonly used is of the form

$$w(t, \tau_n, Y_n) = Y_n \exp\{-(t - \tau_n)/c\}, \tag{2}$$

where c is a parameter that must be estimated.

The flow is a random variable $X(t)$ represented by the continuous-time filtered Poisson process. Moreover, it is generally assumed that $Y_1, Y_2, \ldots$ are independent and identically distributed random variables having an exponential distribution with parameter μ and are also assumed to be independent of the Poisson process $\{N(t), t \geqslant 0\}$. The random variables Y_n are the input process considered as the amount of precipitation.

In this model, the flow has an exponential response to precipitation, and it consists of a coupled Poisson-exponential process that reproduces the occurrence and intensity of effective rainfall. Weiss [3,4] used (2) to model the recession flow, assuming the system to be linear. The random events used in the formation of $X(t)$ result in the occurrence of flooding at random times according to a Poisson process. The choice of an exponential function for the transfer function $h(\tau) := \exp\{-(t - \tau)/c\}$ makes the process $X(t)$ an autoregressive process of order 1, denoted by $AR(1)$, and corresponds to a single linear reservoir; $h(\tau)$ thus determines the recession slope of $X(t)$.

Remark 1.1. A filtered Poisson process is a continuous-time process, whereas an $AR(1)$ process is a discrete-time process. However, in practice, flow values are generally available on a daily basis, so that the filtered Poisson process is discretized.

Most of the efforts in the study of the filtered Poisson process focused on defining the response function of the system, mainly based on physical aspects, and on methods to estimate the parameters; see, for example, Bernier [2], Lawrance and Kottegoda [6], Weiss [3,4], Kelman [9], Koch [10], Konecny [11], Seidou et al. [12] and Murrone et al. [13].

Several other stochastic processes describing the time series of flows have been proposed to model the flow of rivers, for example by Salas et al. [14] and by Dacruz Évora [15]. Other papers have been devoted to analyzing the statistical properties of daily streamflows, for example in Kavvas and Delleur [16]. Recent papers on the use of filtered Poisson processes in hydrology are the ones by Yin et al. [17] and Miyamoto et al. [18]; see also Rajagopalan et al. [19].

Weiss [3,4] commented on the realism of the filtered Poisson process model for daily river flows, compared to other models characterized by Gaussian inputs. The filtered Poisson process is defined by discontinuities at the arrival times of events, followed by exponential decreases. By choosing the response function in (2), the effect on the river flow of the event that occurred at time τ_n is supposed to be maximal at that time instant, and the flow immediately starts to decrease, as shown in Fig. 1; see also Bodo and Unny [5] and Konecny [11].

However, looking at a real hydrograph, we notice that this is not the case. In practice, there is most often a time period during which the flow increases from a minimum to a maximum. Then, it starts to decrease more or less rapidly; see Fig. 2.

The filtered Poisson process is well known to model the hydrograph recessions, but the fact remains that the period of increasing flows is not taken into account in this model with the response function in (2).

To obtain the kind of behavior observed in reality, Lefebvre and Guilbault [7] proposed to use the response function

$$w(t, \tau_n, Y_n) = Y_n (t - \tau_n)^k \exp\{-(t - \tau_n)/c\}, \tag{3}$$

where k is a positive parameter. They showed that this response function indeed leads to a more realistic model.