Applied Mathematical Modelling 39 (2015) 284–296

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Reliability, MTTF and steady-state availability analysis of systems with exponential lifetimes

B. Çekyay [a], S. Özekici [b,*]

[a] Department of Industrial Engineering, Doğuş University, 34722 Acıbadem-İstanbul, Turkey
[b] Department of Industrial Engineering, Koç University, 34450 Sarıyer-İstanbul, Turkey

ARTICLE INFO

Article history:
Received 17 January 2013
Received in revised form 13 February 2014
Accepted 20 May 2014
Available online 2 June 2014

Keywords:
System reliability
Steady-state system availability
Mean time to failure
k-out-of-n standby system
Cold standby redundancy
Warm standby redundancy

ABSTRACT

Our analysis focuses mainly on coherent systems and series connection of k-out-of-n standby subsystems with exponentially distributed component lifetimes. We analyze system reliability, mean time to failure, and steady-state availability as a function of the component failure rates. Our primary objective is provide explicit expressions for these performance measures and obtain various characterizations on their mathematical structures. This primarily involves difference of convex functions which are known to be very useful in the context of optimization problems.

1. Introduction

In this paper, the primary objective is to analyze reliability, mean time to failure (MTTF), and steady-state availability of coherent systems (CS) and series connection of k-out-of-n standby subsystems (SR) with exponentially distributed component lifetimes. The maintenance policy is such that all failed components are replaced by brand new ones only when the whole system fails. Since the lifetimes are exponentially distributed, the system is as good as brand new after a replacement. Moreover, we presume that the repair times depend on the state of the system (number and type of working and failed components) at the time of failure.

We want to point out that this paper came out as a by-product of a research project on the component testing problem of mission-based systems. The component testing problem for any system basically involves the hypothesis testing problem

$$H_0 : R(\lambda) \leqslant R_0 \quad H_1 : R(\lambda) \geqslant R_1, \tag{1}$$

where $R(\lambda)$ is the performance measure of the system (like reliability, MTTF, and steady-state availability) as a function of the unknown component failure rates given by λ. Here, R_1 is a desired acceptable performance level and R_0 is an unacceptable performance level with $R_0 < R_1$. The problem is to design a statistical testing procedure on the system components that satisfies given requirements on the type I and type II error probabilities at minimum cost. In particular, if t_j is the total amount of time for which component j should be tested, then the decision problem is stated as

* Corresponding author. Tel.: +90 (212) 338 1723.
E-mail addresses: bcekyay@dogus.edu.tr (B. Çekyay), sozekici@ku.edu.tr (S. Özekici).

http://dx.doi.org/10.1016/j.apm.2014.05.029
0307-904X/

$$\min \sum_j c_j t_j, \tag{2}$$

subject to the conditions

$$P\{\text{Reject } H_0 | H_0\} \leqslant \alpha, \tag{3}$$

$$P\{\text{Reject } H_1 | H_1\} \leqslant \beta, \tag{4}$$

where c_j is the cost of testing component j per unit time, α and β are the required type I and type II error probabilities respectively. In other words, the problem is to determine the optimal testing durations of the components at minimum cost while the reliability of the testing procedure is at an acceptable level.

Component testing is done when it is very costly or often impossible to test the whole system. Aircrafts used in space missions or nuclear devices are typical examples where various performance measures associated with the devices can be predicted using data on component lifetimes. In the component testing literature, it is generally assumed that component lifetimes are exponentially distributed and one has to find explicit expressions for the performance measure as a function of the unknown component failure rates. Using this explicit structure, one can make a semi-infinite linear programming formulation and solve it by using an efficient algorithm to find the optimal solution. We refer the reader to Altınel et al. [1] and Yamangil et al. [2] for details and examples regarding the component testing problem. The algorithms used in the solution stage use some structural properties like convexity of the performance measure as a function of the failure rates. Our effort in the present setting covers the MTTF and steady-state availability in addition to the reliability of the system, which is the commonly used performance measure in all of the component testing literature. The objective is to find explicit functions for these measures and identify their structural properties that may be used in an optimization context. Another line of research in which our results may be useful concerns Bayesian analysis of reliability systems. In Bayesian applications, the component failure rates are not assumed to be known; rather, they are random variables with some prior distributions. The explicit structure of the reliability and other functions as a function of the failure rates will be helpful in conducting posterior analysis.

The analysis in this paper first focuses on the reliability and MTTF of CS where we obtain difference of convex (DC) representations of these measures. Although this analysis is not mathematically complicated, the DC characterization is very important since optimization models involving DC functions can be solved efficiently as stated in Horst and Thoai [3]. Then, we analyze the reliability and MTTF of SR under three different standby redundancy structures, namely cold (CSR), warm (WSR), and hot (HSR) standby redundancy. We show that the reliability and MTTF of SR are DC, and give explicit DC representations for the reliability and MTTF of HSR assuming that all components in a subsystem are identical. Moreover, it is also shown that the MTTF of CSR is a ratio of posynomials (RP) with positive integer powers. This result is also useful in solving an optimization model including MTTF of an CSR since the natural logarithm of an RP function can be transformed into a DC function, which is good news in solving the optimization problem. Then, we discuss the steady-state system availability for CS and SR, and propose closed-form expressions by using a renewal theoretic approach. Finally, we show that the steady-state availability of CS and SR are both RP with positive integer powers if the repair durations are also exponentially distributed.

There is a huge amount of literature on coherent structures. Most of these papers assume that whenever a component fails, it is repaired and all components are maintained separately. The distribution of the time to failure is analyzed by Barlow and Proschan [4] and Brown [5], and formulas for the interval availability, and the expected number of failures and replacements in a fixed interval are given by Baxter [6]. In our setting, we analyze a different system where failed components wait for the failure of the system to be replaced.

Systems with a k-out-of-n structure attract special attention in the reliability literature because they have a very broad application area. The MTTF for k-out-of-n systems is analyzed by Angus [7], and the mean operating and repair times between two successive breakdowns, the system availability and some mean first-passage times are studied by Iyer [8]. Moreover, Li et al. [9] give formulas for the mean time between failures, mean working time in a failure-repair cycle and mean down time in a failure-repair cycle. In these studies, it is assumed that all lifetimes and repair times are exponentially distributed, there are enough repairmen for all components, and replacement for a component starts immediately after its failure.

The availability and mean time between failures for k-out-of-n systems with M cold standby units that are either identical or non-identical to active components are investigated by Wang and Loman [10]. The availability, the expected up-time, and the expected down-time for a k-out-of-n system with general lifetimes and exponential repair times or vice versa are discussed by Frostig and Levikson [11] using Markov renewal processes. The references listed above assume that the repair of a malfunctioning component starts immediately after its failure. A k-out-of-n system in which failed components are not repaired until system failure is analyzed by Koucký [12], and a closed form reliability formula is derived for a quite general system. Moreover, de Smidt-Destombes et al. [13] analyze the availability of a k-out-of-n system with identical components whose maintenance is initiated when the number of failed components exceeds a critical level. Several efficient algorithms to compute various reliability and availability indices for k-out-of-n systems with arbitrary failure and repair distributions can be found in the paper by Amari et al. [14]. In this paper, we present reliability and mean time to failure results for a model which extends the previous studies to series connection of k-out-of-n subsystems. However, for the sake of a