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A new image segmentation model with local statistical characters based on variance minimization

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A R T I C L E I N F O

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A B S T R A C T

Chan–Vese (CV) model is a promising active contour model for image segmentation. However, CV model does not utilize local region information of images and thus segmentation method based on CV model cannot achieve good segmentation results for complex image with some in-homogeneity intensities. To overcome the limitation of CV model, this paper presents a new type of geometric active contour model using the strategy of variance minimization of image and introduced local statistics in the new energy formulation. The proposed model not only considers the first and second order moments of objective image statistical measurements, but also regularizes the level set function by incorporating the distance penalized energy function. The major contributions of this paper conclude two aspects. One is the new energy function based on variance minimization and another is the introduction of the local weighted averaging. In this paper, we get the local weighted averaging by the pieces smooth approximation through Gaussian convolution. Experimental results demonstrate that the proposed approach is effective in image segmentation, especially for the image with in-homogeneity intensity.

1. Introduction

In the past years, geometric active contour models (ACMs) [1] have been widely applied to image segmentation, de-noising and object tracing tasks. It is known that ACMs, based on curve evolution and level set theories, can deal with topological change regions automatically in image segmentation. In general, ACMs can be roughly classified into two categories, namely the edge-based models [1–5] and the region-based models [6–10]. The edge-based models drive the curve to object boundary using image gradient information. As a result of avoiding the local minimization of optimization, edge-based models seriously depend on the initial curves. While for region-based models, they make use of region information to control the curve evaluation. Therefore, region-based models possess many advantages over the edge-based models. On the one hand, region-based models are less sensitive to the image noise and robust to the images with weak edges or without edges because they exploit the statistical moment information inside and outside the evolution contours, and on the other hand the region-based models are significantly less sensitive to the initial contour location and own the ability for efficiently detecting the exterior and interior boundaries simultaneously. One of the most popular region-based models is the Chan–Vese model [6] derived from the Mumford–Shah segmentation techniques [8], which has been successfully applied to binary phase image segmentation. Additionally, Caselles [1] proposed the geodesic active contour (GAC) model and Malladi [11] proposed

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the edge and gradient induced active contour models. Paragios et al. [12] replaced the vector field force with the well-known gradient vector flow (GVF) introduced by Xu et al. [13] to increase the capture range. Unfortunately, three gradient-based models mentioned above cannot deal with complex images, such as images with noise or in the low-contrast. At the same time, boundaries of images with noise and low contrast are ambiguous, so they cannot be detected with gradient information. By utilizing the region information instead of the image gradient information, Chan–Vese model [6] is more suitable and accurately than gradient-based (edge-based) geometric active contour model. However, since the convergence of the Chan–Vese model depends on the homogeneity of the segmented objects, Chan–Vese model cannot provide satisfactory segmentation results for in-homogeneity images such as carpal bones and knee bones images.

It is known that the variance information of statistical measurement of image represents the fluctuation magnitude of image intensities and the local weighted averaging can depict the image gray value more suitable. This means that a charming segmentation can be obtained by minimizing the associated intensity variance of the image and adopting the local mean instead of the global mean. The local weighted averaging can be obtained by computing the convolution between the image gray values and the kernel function. So, by using the technique of variance minimization of images and the trick which is utilizing the local weighted averaging, this paper proposes a new type of geometric active contour model for accurately segmenting an image with in-homogeneity intensities. The idea of the proposed model is derived from the Chan–Vese model [6], density distance model [14] and Efficient Segmentation of Piecewise Smooth Images [15]. Especially, our proposed model fully considers the first and second order moments of objective image statistical measurements, and incorporates the distance penalized energy function to regularize the level set function. In addition, we choose the Gaussian function as the kernel function in this paper. Due to the introduction of Gaussian function, we have to consider the variance parameter σ of the Gaussian function. As $\sigma \to \infty$, the local mean would equal to the global mean. We also obtain our desire segment results by tuning the value of the variance parameter σ. It is no doubt that tuning the value of the variance parameter σ provides more flexible for image segmentation. Experimental results demonstrate that the proposed approach is effective for medical image segmentation.

The rest of this paper is organized as follows: Section 2 introduces the related works. Our new model is proposed in Section 3. The experimental results are shown in Section 4. Finally, we draw the conclusions in Section 5.

2. Related works

We will introduce Chan–Vese model, the density distance model and piecewise smooth approximation through Gaussian convolutions in this section.

2.1. Chan–Vese model

Chan–Vese model derives from the two-value image segmentation, which utilizes mean values to divide the image into the targets and background. For a given image $u_0(x,y)$ in domain Ω, Chan–Vese model is formulated by minimizing the following energy functional

$$E(C,c_1,c_2) = \lambda_1 \int_{inside(C)} |u_0(x,y) - c_1|^2 dxdy + \lambda_2 \int_{outside(C)} |u_0(x,y) - c_2|^2 dxdy + \mu \int_{\Omega} |\nabla H(\phi(x,y))| dxdy, \quad (1)$$

where C represents the moving contour, $u_0(x,y)$ is the original image, $\lambda_1, \lambda_2 > 0, \mu \geqslant 0$ are the weights of each corresponding item, c_1 and c_2 are the mean of gray values inside and outside the moving contour. ∇ represents the operator of gradient. $\phi(x,y)$ is level set function and Heaviside functional [6,16] is

$$H(\phi) = \begin{cases} 1 & \phi \geqslant 0 \\ 0, & \phi < 0 \end{cases}. \quad (2)$$

c_1 and c_2 can be calculated by

$$\begin{cases} c_1 = \frac{\int_{\Omega} u_0(x,y) H(\phi(x,y)) dxdy}{\int_{\Omega} H(\phi(x,y)) dxdy} \\ c_2 = \frac{\int_{\Omega} u_0(x,y)(1-H(\phi(x,y))) dxdy}{\int_{\Omega}(1-H(\phi(x,y))) dxdy} \end{cases} \quad (3)$$

It is obvious that the value of energy function of Chan–Vese model is non-negative. At the same time, the value of the first two items in $E(C,c_1,c_2)$ would be zero when the active contour C coincides to the true boundary of the object in two-value image. The first two items in Eq. (1) are named the external energy, which are employed to drive the contour to the correct position. The last item is named the internal energy, which is used to regularize the geometric properties of the moving contour.

In order to obtain the corresponding derivate of the Heaviside function, $H_\varepsilon(\phi)$ can be replaced by

$$H_\varepsilon(\phi) = \frac{1}{2}\left(1 + \frac{2}{\pi}\arctan\left(\frac{\phi}{\varepsilon}\right)\right), \quad (4)$$