Applied Mathematical Modelling 39 (2015) 3357–3365

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

A Mayer-type optimal control for multivalued logic control networks with undesirable states

Yang Liu [a], Hongwei Chen [a,c], Bo Wu [a,b,*], Liangjie Sun [a]

[a] College of Mathematics, Physics and Information Engineering, Zhejiang Normal University, 321004 Jinhua, China
[b] Graduate School, Zhejiang Normal University, Jinhua 321004, China
[c] Department of Mathematics, Southeast University, Nanjing 210096, China

ARTICLE INFO

Article history:
Received 21 June 2013
Received in revised form 9 April 2014
Accepted 17 November 2014
Available online 3 December 2014

Keywords:
Optimal control
Multivalued logical (control) network
Semi-tensor product
Systems biology

ABSTRACT

A fundamental problem for all dynamical control systems is to determine a control that is optimal in some sense. Based on semi-tensor product, we consider a Mayer-type optimal control problem for multi-valued logical control networks with time delays in states. A necessary condition for a control to be optimal is derived while avoiding a set of forbidden states. Examples are provided to illustrate the effectiveness of the obtained results.

1. Introduction

Resent years witnesses an increasing research interest in the study of complex networks which provide a simple and proper model to describe a wide range of intricate natural systems and phenomena [1,2] One kind of complex networks called genetic regulatory networks (GRNs) has been a new research area in the biological science and have attracted great attention over the last few years. There have been a number of attempts to model gene regulatory networks, such as Bayesian network, differential equations, Boolean networks [3]. In particular, Boolean networks have been successfully used to characterize gene regulatory networks where the ON (OFF) state corresponds to the transcribed (quiescent) state of the gene.

Boolean network is the simplest logical dynamic system which was proposed by Kauffman for modeling complex and nonlinear biological systems, see [3,4]. Since then, it has been a powerful tool in describing, analyzing, and simulating the cell networks, see [5,6]. The analysis and control of Boolean network is a challenging problem. So far, there are only few results on it because of the shortage of systematic tool to deal with logical dynamic systems, see [5]. Recently, a new matrix product which was called the semi-tensor product (STP), was provided to convert a logical function into an algebraic function and convert the logical dynamics of Boolean networks into standard discrete-time dynamics. Based on STP, there have been many interesting studies on analyzing and synthesizing Boolean (control) networks, see [7,8].

It is well known that time delay phenomenon is very common in real world [9–11], and very important in analysis and control for dynamic systems [12–15]. Time delay behaviors happen frequently in biological and physiological systems [16,17]. In [17], a delay-dependent sufficient condition was obtained in the linear matrix inequality (LMI) form such that delayed stochastic genetic regulatory networks are globally asymptotically stable in the mean square. In [18], a model for

* Corresponding author at: Graduate School, Zhejiang Normal University, Jinhua 321004, China. Fax: +86 57982298897.
E-mail address: wubo_502@163.com (B. Wu).

http://dx.doi.org/10.1016/j.apm.2014.11.028
0307-904X/

GRNs with time delays was proposed and nonlinear properties of the model in terms of local stability and bifurcation was analyzed. In [19], genetic networks with delays and stochastic perturbations have been studied and sufficient conditions of stability were derived in terms of linear matrix inequalities (LMIs). In [20], sufficient conditions have been derived to ensure the global exponential stability of the discrete-time GRNs with delays.

On the other hand, multivalued networks appear in some complex systems, for instance in chemical reactions [21], cognitive sciences [22], etc. In [23], a class of cellular automata (CA) have been studied by using algebraic technique. The global properties of CA are determined by a fixed polynomial. All the results give by [23] can be used for p-valued CAs, where p is a prime integer. In [24], the stability and stabilization of a class of multivalued logical networks have been discussed. In [25], the controllability of multivalued logical control network (MLCN) via two kinds of controls has been discussed. Hence, study the dynamics of the multivalued networks with time delays is meaningful and challenging.

Systematic analysis of biological systems is important in systems biology. Controllability and optimal control problem are two interesting topic on system control theory [26–31]. In [32], the controllability of probabilistic Boolean control networks has been studied in terms of transition probability matrices. In [26], the numerical solution of optimal control problems for probabilistic Boolean networks has been investigated using dynamic programming. In [27], the optimal controls for Boolean control networks have been revealed over a certain cycle. A Mayer-type optimal control problem for Boolean control network with multi-input and single-input has been studied in [28,29]. For biological systems, as some states may correspond to unfavorable or dangerous situations, there would be certain forbidden states. The controllability of Boolean control networks with a set of forbidden states has been investigated via Perron–Frobenius theory in [30] and the controllability of Boolean control networks with impulsive disturbances have been studied in [33]. The output controllability problem of temporal Boolean networks has been considered in [34]. However, Akutsu et al. [35] demonstrated that control problems for Boolean control networks were in general NP-hard.

To the best of the authors knowledge, there is no result available yet on a Mayer-type optimal control problem for a MLCN with time delay, while avoiding a set of forbidden states. The delayed system considered here is a special class of higher-order multivalued logic control networks which can be rewritten by a MLCN without delay by using the first algebraic form of the network. Then one can analyse a delayed MLCN by the corresponding MLCN. However, if the first algebraic form is used, the dimension of network transition matrix depending on the number of logical variables will be much larger which would make computation cost much higher. Hence, we investigate the delayed MLCNs directly. When undesirable set exists, the optimal control analysis become more complicated. Based on a new MLCN with respect to the undesirable set, we obtain a necessary condition for a control to be optimal and provide a kind of Pontryagin maximum principle (PMP) for the MLCN.

2. Preliminaries

For simplicity, we first give some notations. Denote $\mathbb{N}$ and $\mathbb{N}^+$ as the sets of nonnegative integer and positive integer, respectively. Denote $M_{m\times n}$ as the set of all $m \times n$ matrices. The delta set $\Delta_k := \{\delta_k^i | i = 1,2,\ldots,k\}$, where δ_k^i is the ith column of identity matrix I_k with degree k. A matrix $A \in M_{m\times n}$ is called a logical matrix if the columns set of A, denoted by $Col(A)$, satisfies $Col(A) \subset \Delta_m$. The set of all $m\times n$ logical matrices is denoted by $\mathcal{L}_{m\times n}$. Assuming $A = [\delta_m^{i_1}, \delta_m^{i_2}, \ldots, \delta_m^{i_n}] \in \mathcal{L}_{m\times n}$, we denote it as $A = \delta_m[i_1, i_2, \ldots, i_n]$.

We recall the concept of STP of matrices. Consider an $m\times n$ matrix A and a $p \times q$ matrix B, a STP of A and B is denoted by

$$A \ltimes B = (A \otimes I_{\alpha/n})(B \otimes I_{\alpha/p}),$$

where α is the least common multiple of n and p, and $\otimes$ is the tensor (or Kronecker) product. When $n = q = 1, A\ltimes B = A \otimes B$. When $n = p, A\ltimes B = AB$. So it is a generalization of the conventional matrix product. In this paper, "$\ltimes$" is omitted and the matrix product is assumed to be the semi-tensor product as in [7,8].

Remark 2.1. The matrix $W_{[n,m]}$ seems to be the matrix of the linear map $W : (\mathbb{R}^n \otimes \mathbb{R}^m) \to (\mathbb{R}^m \times \mathbb{R}^n)$, defined by $W(x \otimes y) = y \otimes x$, where $\otimes$ denotes the tensor product. In a similar way, the semi-tensor product of an $m \times n$ matrix A and $p \times q$ matrix B is the matrix of the following linear map $L : \mathbb{R}^q \otimes \mathbb{R}^{\alpha/p} \to \mathbb{R}^m \otimes \mathbb{R}^{\alpha/n}$, where α is the smallest common multiple of n and p, such that the following holds: if $x \otimes y \in \mathbb{R}^q \otimes \mathbb{R}^{\alpha/p}$, then let $z = Bx \otimes y \in \mathbb{R}^p \otimes \mathbb{R}^{\alpha/p}$ which can naturally be identified with an element $\mathbb{R}^\alpha$. Any element of $\mathbb{R}^\alpha$ can naturally be identified with $k \otimes l \in \mathbb{R}^n \otimes \mathbb{R}^{\alpha/n}$. Hence, $z = Bx \otimes y$ can be written as $k \otimes l$ and then $L(x \otimes y) = Ak \otimes l$.

An $mn \times mn$ matrix $W_{[m,n]}$ is called swap matrix, if it is constructed in the following way: label its columns by $(11, 12, \ldots, 1n, \ldots, m1, m2, \ldots, mn)$ and its rows by $(11, 21, \ldots, m1, \ldots, 1n, 2n, \ldots, mn)$. Then its element in the position $((I,J),(i,j))$ is assigned as

$$w_{(I,J),(i,j)} = \delta_{i,j}^{I,J} = \begin{cases} 1, & I = i \text{ and } J = j, \\ 0, & \text{otherwise}. \end{cases}$$

When $m = n$, we briefly denote $W_{[n]} = W_{[m,n]}$. Furthermore, for $X \in \mathbb{R}^m$ and $Y \in \mathbb{R}^n, W_{[m,n]} \ltimes X \ltimes Y = Y \ltimes X, W_{[n,m]} \ltimes Y \ltimes X = X \ltimes Y$.

A k-valued logical domain, denoted by $\mathcal{D}_k$, is defined as $\mathcal{D}_k := \{T = 1, \frac{k-2}{k-1}, \ldots, \frac{1}{k-1}, F = 0\}$. To use matrix expression we identify each element in $\mathcal{D}_k$ with a k-dimensional vector as $T = 1 \sim \delta_k^1, \frac{k-2}{k-1} \sim \delta_k^2, \ldots, F = 0 \sim \delta_k^k$. Hence, $\mathcal{D}_k$ is equivalent to