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Groundwater flow in hillslopes: Analytical solutions by the theory of holomorphic functions and hydraulic theory

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ABSTRACT

Three 2-D steady Darcian flows in an aquifer with a subjacent confining layer of a non-constant slope or a bedding inconformity are studied by two models: a potential theory (conformal mappings, the inverse boundary-value problem method, and the theory of R-linear conjugation) and hydraulic approximation. First, flow over a corner, whose vertex is either a stagnation point or point of infinite Darcian velocity, is analysed as a transition from one "normal" regime upstream to another downstream. The hodograph domain is a circular triangle, which is mapped onto a complex potential strip via an auxiliary half-plane. Parametric equations (backwater curves) for the phreatic surface are obtained. For the same flow problem, a depth-averaged 1-D nonlinear ODE for the thickness of the saturated zone (a generalized Dupuit–Fawer model) is numerically solved showing a perfect match with the potential (2-D) solution. Second, a non-planar aquifuge boundary is reconstructed as a streamline, along which an additional "control" boundary condition holds in the form of pore pressure as a function of an auxiliary variable (a relation between the hydraulic head and vertical Cartesian coordinate). The free surface is found in terms of Cauchy's integrals for the Zhukovskii function, with explicit integrations for selected "controls". Third, a confined flow in a two-layered aquifer having a lens-type semi-circular inclusion in the subjacent stratum and incident velocity parallel to the interface between two aquifers is examined. The conjugation conditions along all four boundaries, across which the hydraulic conductivity jumps, are exactly met. The three velocity fields are explicitly presented, with examination of the flow net, including separatrices ("capture zone" boundaries), demarcating suction/barriering of the lens, and evaluation of the lens-induced cross-flow (commingling) between the two strata.

1. Introduction and hydrogeological motivation

There is a constant yearning for all that is unconfined.
F. Holderlin *Mnemosyne*.

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Unconfined aquifers in groundwater hydrology represent an interesting object for modeling because of a free (phreatic) surface and nonlinear boundary conditions there (see e.g., [1,2]). Mathematically, the corresponding free boundary problems are similar to ones in open channel flows [3,4]. Groundwater motion is analyzed in catchment-scale reconnaissance models or in regular annual assessment of aquifers' resources by either a hydraulic (ODE) or hydrodynamic (PDE) theoretical description ([5,6], hereafter abbreviated as PK62). The former is called the Dupuit–Forcheheimer (DF) model (see its recent generalizations in the so-called Dupuit–Fawer approximation, [7]), which in steady regimes and homogeneous rock requires solving a BVP for a second-order ODE. The latter, the potential theory -PT, calls for solving a BVP for Laplace's equation.

In arid climates with little recharge from the vadose zone to the phreatic surface, the main factor controlling its shape and locus in a relatively homogeneous aquifer is the subjacent bedrock whose geometry is commonly inferred from geological data. In the study area (Northern Oman), for which our mathematical models are developed, the geology is complex. It ranges from the Precambrian basement rocks, mainly phyllites and slates, at the bottom of the succession occupying the core of North Oman Mountains (NOM) to karstified carbonate rocks (Hajar Supergroup HSG) at the elevated areas to fractured ophiolitic sequence overlain by porous medium of Tertiary limestones and Quaternary alluvium gravel at the top of the geologic section.

The vertical cross section in Fig. 0 illustrates the field relations between the different geological units from the elevated area of NOM to the Gulf coast. The boundary between the carbonates and the ophiolites is controlled by major fault system along which several springs are originating. The Tertiary limestones and alluvium is thickening from elevated area downstream to exceed 300 m at the lower plain coastal areas and comprises the main source for groundwater production in vast areas of Northern Oman. The alluvium is deposited under alluvial and deltaic depositional environment, originates at the piedmonts of NOM and extends into the plain areas forming fan structures. The ophiolites bounding the alluvium have irregular surface ranging from steep at the NOM piedmonts to nearly planar at the coastal areas. The alluvium is predominantly composed of gravels, driven from ophiolites weathering, which vary in shape and size and mixed with fines. The portion of fines increases from the proximal to the distal part resulting in the formation of clayey silt lenses with low permeability imbedded within the alluvium. Groundwater motion there led to the precipitation of $CaCO_3$ and SiO_2 within the pores which enhances cementation and diagenesis processes in the lower alluvium layer leading to the formation of a cemented gravely unit at the bottom. Therefore, the hydraulic conductivity of this unit is much less than that of the overlying unit that is predominantly composed of loose gravels. Although it is formed of similar geological material, the alluvium forms two hydrogeological units (aquifer–aquifer or aquifer–aquifuge) owing to variation in hydraulic properties due to cementation and diagenesis variation.

The recent studies of the elevation of the phreatic surface (water table) in Fig. 0 revealed its puzzling spatial variability detected in direct borehole observations and reconstructed geophysically (mostly by TDEM) (see, e.g., [8–11]). West–East decreasing slopes of the interfaces between different hydrogeological units (ophiolite–carbonate–cemented gravel–gravel) in Fig. 0 and bedrock troughs (lenses) filled with sediments of permeability contrasting with the main surrounding rock, have been found both geophysically (by seismics) and from exploratory drilling. Generally, as Fig. 0 illustrates, the water table slope is steep in the mountains and relatively mild in the valley part of the catchment (see also [12,9]). The degree of this steepness and position of the water table is vital in Oman where groundwater is the only resource for agriculture and main resource in other sectors of economy. Both traditional (falaj) and modern (tube well) water supply schemes tap unconfined aquifers by either intercepting the spring discharge or relatively shallow water table in Fig. 0.

In this paper we answer the following questions: (a) How to accurately describe groundwater dynamics in aquifers with non-planar bedrock as in Fig. 0? (b) When a relatively simple DF model is suitable and what is its error as compared with the PT?

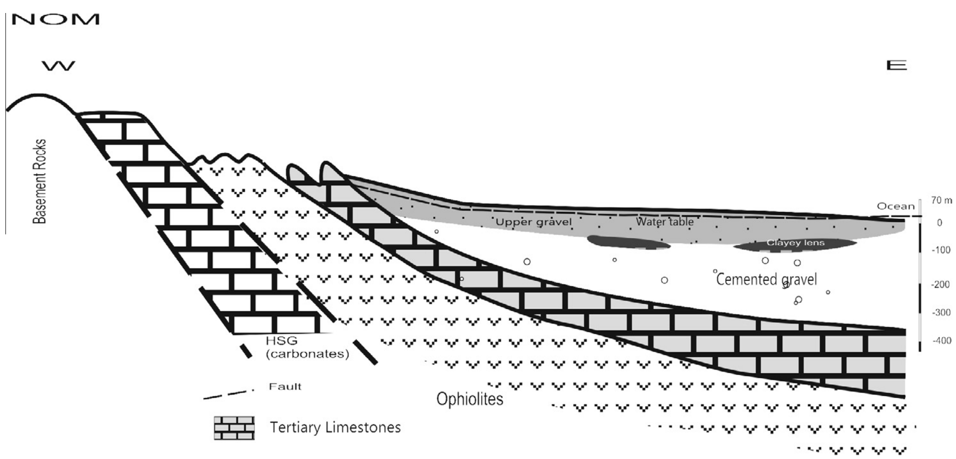

Fig. 0. Typical hydrogeological cross-section of a coastal aquifer in Northern Oman.