Applied Mathematical Modelling 39 (2015) 3434–3447

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Modeling water uptake by a root system growing in a fixed soil volume

Jorge Luis Blengino Albrieu [a,*], Juan Carlos Reginato [a], Domingo Alberto Tarzia [b]

[a] Departamento de Física, Facultad de Ciencias Exactas Físico-Químicas y Naturales, Universidad Nacional de Río Cuarto, Ruta 8 km 601, X5804BYA Río Cuarto, Córdoba, Argentina

[b] Departamento de Matemática – CONICET, Facultad de Ciencias Empresariales, Universidad Austral, Paraguay 1950, S2000FZF Rosario, Argentina

A R T I C L E I N F O

Article history:
Received 28 February 2012
Received in revised form 23 September 2014
Accepted 20 November 2014
Available online 6 December 2014

Keywords:
Moving boundary
Water uptake
Plant root growing

A B S T R A C T

The water uptake by roots of plants is examined for an ideal situation, with an approximation that resembles plants growing in pots, meaning that the total soil volume is fixed. We propose a coupled water uptake–root growth model. A one-dimensional model for water flux and water uptake by a root system growing uniformly distributed in the soil is presented, and the Van Genuchten model for the transport of water in soil is used. The governing equations are represented by a moving boundary model for which the root length, as a function of time, is prescribed. The solution of the model is obtained by front-fixing and finite element methods. Model predictions for water uptake by a same plant growing in loam, silt and clay soils are obtained and compared. A sensitivity analysis to determine relative effects on water uptake when system parameters are changed is also presented and shows that the model and numerical method proposed are more sensitive to the root growth rate than to the rest of the parameters. This sensitivity decreases along time, reaching its maximum at 30 days. A comparison of this model with a fixed boundary model with and without root growth is also made. The results show qualitative differences from the beginning of the simulations, and quantitative differences after 10 days of simulations.

1. Introduction

In the development of a theory to describe plant water uptake, electrical analogues of the system have been used for analysis [1–3]. The analogues are based on the assumption that rooting patterns are uniform and constant in each soil layer. Steady flow is presumed in both the soil and the plant over the period of calculation. In this approach the plant water potentials are primarily the result of an imposed value of transpiration rate and its variations. Later papers [4–6] have presented detailed reviews on plant water uptake. In those papers the Richards equation is used, with a sink term. Another approach is to model the water movement and uptake over large areas, using individual plant [7], or global behavior [8]. A microscopical approach has also been proposed [9], where the total water uptake is calculated based on using a constant value for the entire rooting profile. In more recent papers [10–12] the root growth has been taken into account, still using a fixed domain. The root growth is inscribed on a domain that is not a function of time. Some other papers about nutrient uptake consider root growth and instantaneous coupling with the nutrient flux by using a variable domain approximation [13]. In this last model [13] a variable root length, and consequently, a variable available volume of soil to each root of a root system is

* Corresponding author.
E-mail address: jblengino@exa.unrc.edu.ar (J.L. Blengino Albrieu).

http://dx.doi.org/10.1016/j.apm.2014.11.042
0307-904X/

considered using a moving boundary model. In this model the root system is uniformly distributed in the soil and the variation of available soil volume per unit of root length is modeled by a moving boundary. The approach presented here is based on that in [13]. In the proposed model plants growing in controlled conditions, as in a growth chamber, are assumed. A constant temperature and evapotranspiration rate is presumed. In this situation, the water potential at the root surface is determined by the soil water potential, and consequently determines water uptake by the growing root system. The proposed model considers an uniform root water uptake for all the root system. The goal of this paper is to present a simplified model of water uptake coupled with a growing root system and analyze the influence of system parameters on water uptake using typical values.

2. Model

Darcy's law describes the flow of water on a porous unsaturated medium as

$$\vec{J}(\vec{r},t) = -K(\Psi(\vec{r},t))\vec{\nabla}\Psi(\vec{r},t), \tag{1}$$

with $\vec{J}$ [cm^3/cm^2 s] the water flux per surface unit at position $\vec{r}$ [cm] at time t [s], K [cm/s] the soil water conductivity and Ψ [cm] the soil water potential.

The corresponding continuity equation (mass conservation) is given by

$$-\nabla\cdot\vec{J} = \frac{\partial\theta}{\partial t}, \tag{2}$$

with θ [cm^3/cm^3] the soil water content per unit of volume, with the approximation of only radial flux the transport equation results

$$\frac{\partial}{\partial t}[\Psi(r,t)] = -\frac{1}{rC(\Psi(r,t))}\frac{\partial}{\partial r}\left[rK(\Psi(r,t))\frac{\partial}{\partial r}[\Psi(r,t)]\right], \tag{3}$$

where r is the cylindrical radial coordinate, and

$$C(\Psi(r,t)) = \frac{d\theta}{d\Psi}(\Psi(r,t)) \tag{4}$$

is the differential capacity of water [cm^{-1}].

The soil water constitutive relations for K, θ, and C, as functions of Ψ, are the ones proposed by Van Genuchten [14], and consist of the expressions given by:

$$K(\Psi) = K_s\frac{\left\{\left[1+\left(\frac{\Psi}{\Psi_e}\right)^n\right]^m - \left(\frac{\Psi}{\Psi_e}\right)^{n-1}\right\}^2}{\left[1+\left(\frac{\Psi}{\Psi_e}\right)^n\right]^{m(p+2)}}, \tag{5}$$

$$\theta(\Psi) = \left[1+\left(\frac{\Psi}{\Psi_e}\right)^n\right]^{-m}(\theta_s-\theta_R)+\theta_R, \tag{6}$$

$$C(\Psi) = (\theta_s-\theta_R)\frac{1-n}{\Psi_e}\left[1+\left(\frac{\Psi}{\Psi_e}\right)^n\right]^{-m-1}\left(\frac{\Psi}{\Psi_e}\right)^{n-1}, \tag{7}$$

where K_s [cm/s] is the saturated soil conductivity, θ_s [cm^3/cm^3] is the saturated soil water content, θ_R [cm^3/cm^3] is the residual water content, Ψ_e [cm], p[1] and n[1] are experimental coefficients, and $m = 1 - 1/n$.

We presume that the water potential does not change with soil depth, the soil does not evaporate, and laboratory conditions, like light and temperature are maintained constant. The root density is homogeneous on the soil, the total volume is fixed (as in pots), therefore the soil volume per unit of root length is decreasing as in Fig. 1. Based on these assumptions, and taking into account the root length density as a function of time (t) the following moving boundary model in cylindrical coordinates [9,13] is proposed

$$\frac{\partial}{\partial t}[\Psi(r,t)] = -\frac{1}{rC(\Psi(r,t))}\frac{\partial}{\partial r}\left[rK(\Psi(r,t))\frac{\partial}{\partial r}[\Psi(r,t)]\right], \tag{8}$$

$$\Psi(r,0) = \phi(r), \tag{9}$$

$$H(\Psi(R(t),t),t) = 2\pi R(t)K(\Psi(R(t),t))\frac{\partial\Psi}{\partial r}(R(t),t), \tag{10}$$

$$G(\Psi(s_0,t)) = -2\pi s_0 K(\Psi(s_0,t))\frac{\partial\Psi}{\partial r}(s_0,t), \tag{11}$$

$$R(t) = R_0\sqrt{\frac{l_0}{l(t)}}, \tag{12}$$