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Spherical data fitting by multiscale moving least squares ☆

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ABSTRACT

This paper focuses on the multiscale moving least squares approximation scheme on the unit sphere, where the scale depends on the current evaluation points. The scheme is constructed by using a sequence of scaled weight functions, and is a little different from the classical moving least squares approximation on the sphere, which can be obtained by restricting compactly supported radial basis functions in $\mathbb{R}^3$ to $\mathbb{S}^2$. More precisely, a multiscale moving least squares (MMLS) algorithm, in which the corresponding scale is changing with the associated given point set, is proposed. In addition, the convergence analysis for the multiscale scheme and some numerical experiments to illustrate the theoretical results are given.

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1. Introduction

In recent years, scattered data approximation on the sphere $\mathbb{S}^2$ has been an active study area because of its obvious applications to astrophysics, meteorology, geodesy, geophysics and other areas (see [1–3]). Amongst approaches for reconstructing a continuous function on $\mathbb{S}^2$ from a finite number of scattered data, many authors have used spherical polynomials or spherical radial basis functions (see [4–15,3]), and we must remark that this list is far from being complete.

Moving least squares approximation, which was first introduced to scattered data approximation on $\mathbb{R}^n$ several decades ago (see [16,17]), is actually a scheme to approximate target functions by using polynomials. In fact, the Shepard's interpolation method (see [18]) is a special case of moving least squares approximation. In addition, moving least squares acts as a so-called meshless method, which plays an important role in the numerical treatment of partial differential equations (PDEs) (see [19]). In [20], Wendland used results in Euclidean 3-space and then restricted to points on the 2-sphere (see [21]). In [21], the highlight is that moving least squares approximation process involves the local polynomial reproduction property, which is the key ingredient in deriving error estimates. For every point x and the function value $f(x)$, the main idea of moving least squares approximation is to solve a locally weighted least squares problem, where the local continuous weight function is the scaled version of a compactly supported function (see [3]). This seems to be a lot expensive, but it will turn out to be a very efficient method. Because, in many applied areas one is only interested in a few function values. For such applications the moving least squares approximation scheme is much more attractive, because it is not necessary to set up and solve a large linear system compared with the method of spherical radial basis functions (SBFs) interpolation.

We should note the fact that the scale index δ remains the same everywhere in classical moving least squares approximation, which reflects the local approximation by using the local continuous weight function. However, as we all know, the geophysical data typically occur at many different length scales: for example, the topography of the Sahara Desert varies

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slowly, while that of the Himalayas varies rapidly. To accommodate the different length scales, Le Gia, Sloan and Wendland (see [22,23]) proposed a multiscale approximation scheme on $\mathbb{S}^n$ in a creative way, as well as in general bounded domains (see [24]).

Specifically, we should pay attention to the fact that, as an approach by using SBFs rather than spherical polynomials, the multiscale approximation scheme is generated from a single underlying radial basis function $\Psi(x) = \rho(|x|) : \mathbb{R}^{n+1} \longrightarrow \mathbb{R}$ but using a sequence of scales $\delta_1, \delta_2, \ldots$ with limit zero, where the corresponding scale becomes smaller as the point set becomes denser. In [22], Le Gia, Sloan and Wendland gave a multiscale analysis in Sobolev spaces on $\mathbb{S}^n$, where they established a convergence theorem for the multiscale scheme for a smooth target function. Moreover, in [23], Gia, Sloan and Wendland embedded the smooth SBFs in a larger Sobolev space generated by a less smooth kernel, and still used the multiscale scheme associated with the smooth SBFs to approximation the rougher target function from a larger Sobolev space.

Back to the issue about moving least squares approximation, with the help of multiscale scheme for SBFs approximation, we should let δ depend on the current evaluation points, precisely, when the given point x lies in a domain with a high data density, the corresponding scale δ would be chosen small, if there are only few points around x, one has to choose δ rather large. In this paper, we propose the multiscale moving least squares approximation scheme, where the scale index δ associated with the given point sets will change step by step. For the unknown function value $f(x)$, the scheme give the associated pointwise approximator. In addition, we put forward the MMLS algorithm and conduct the convergence analysis for multiscale moving least squares approximation. During the process, we must remark that, we make no assumption that the corresponding point sets at different steps are nested. Finally, we give some numerical experiments to illustrate that the multiscale moving least squares approximation scheme is effective.

This paper is organized as follows. In Section 2, after reviewing the necessary background on spherical harmonics, we state function spaces relevant to this paper, as well as the point sets and geometry on the sphere. In Section 3, moving least squares approximation is reviewed and the associated MLS algorithm is established in order to be useful in the following section. We propose the multiscale moving least squares approximation scheme in Section 4, where the procedure is stated step by step. Importantly, we put forward the MMLS algorithm, which returns the associated pointwise approximator to the original function value. Section 5 is devoted to giving the convergence analysis for multiscale moving least squares approximation, of which the highlight is a Jackson-type inequality for a sufficiently smooth function (see Lemma 5.1). In Section 6, we conduct numerical experiments to demonstrate the effectiveness of multiscale moving least squares approximation scheme.

In this paper, we adopt the following convention regarding symbols. Let $C, C', C_i, \ldots$ be positive constants, where i is either a positive integer or a variable on which C depends only. Their values may be different at different occurrence even within the same formula.

2. Preliminaries

2.1. Spherical harmonics

Let $\mathbb{S}^2$ be the unit sphere embedded in the Euclidean space $\mathbb{R}^3$, i.e.,

$$\mathbb{S}^2 := \{x := (x_1, x_2, x_3) \in \mathbb{R}^3 : x_1^2 + x_2^2 + x_3^2 = 1\}.$$

For integer $l \geqslant 0$, the restriction to $\mathbb{S}^2$ of a homogeneous harmonic polynomial with degree l is called a spherical harmonic of degree l. The class of all spherical harmonics with degree l is denoted by $\mathcal{H}_l$, and it is well know that spherical harmonics of different degrees are orthogonal with respect to the $L_2(\mathbb{S}^2)$ inner product

$$\langle f, g \rangle := \int_{\mathbb{S}^2} f(x) g(x) d\omega(x),$$

where $d\omega$ denotes surface measure on $\mathbb{S}^2$. Hence, if we choose an orthogonal basis $\{Y_{l,k} : k = 1, \ldots, 2l+1\}$ for each $\mathcal{H}_l$, then the set $\{Y_{l,k} : l = 0, 1, \ldots,\ k = 1, \ldots, 2l+1\}$ is an orthogonal basis for $L_2(\mathbb{S}^2)$. The class of all spherical harmonics with total degree $l \leqslant L$ is denoted by $\mathcal{P}_L$. Of course, $\mathcal{P}_L = \bigoplus_{l=0}^{L} \mathcal{H}_l$, and the dimension of $\mathcal{H}_l$ is $2l+1$ and that of $\mathcal{P}_L$ is $(L+1)^2$.

The Laplace–Beltrami operator on $\mathbb{S}^2$ is defined by (see [25,26])

$$\Delta f := \sum_{i=1}^{3} \frac{\partial^2 g(x)}{\partial x_i^2}\bigg|_{|x|=1}, \quad g(x) := f\left(\frac{x}{|x|}\right),$$

where $|x| = \sqrt{x_1^2 + x_2^2 + x_3^2}$.

The well known addition formula is given by (see [25])

$$\sum_{k=1}^{2l+1} Y_{l,k}(x) Y_{l,k}(y) = \frac{2l+1}{4\pi} P_l(x \cdot y), \tag{1}$$

where P_l is the Legendre polynomial with degree l and dimension three, which is normalized such that $P_l(1) = 1$, and satisfies the orthogonality relation (see [25])

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