Applied Mathematical Modelling 39 (2015) 3481–3495

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Modeling wave propagation in moderately thick rectangular plates using the spectral element method

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ARTICLE INFO

Article history:
Received 16 January 2013
Received in revised form 6 July 2014
Accepted 20 November 2014
Available online 9 December 2014

Keywords:
Spectral element method
Vibration
Impact
Dynamic
Moving load
Plate

ABSTRACT

This paper presents development of the spectral element method (SEM) to specify natural frequencies and dynamic response of moderately thick rectangular plates under impact and moving loads. To solve differential equations of moderately thick plate, the displacement field has been expressed in the frequency domain using Fast Fourier Transformation (FFT) algorithm while considering simple boundary conditions for two parallel edges. Closed-form solutions have been derived for the differential equations in frequency domain. Deriving exact shape functions of plate in frequency domain, the dynamic solution in time domain has been calculated using Inverse Fast Fourier Transformation (IFFT). In this study, natural frequencies for moderately thick plates with variable and constant thicknesses have been calculated and compared to the past research results. Mode shapes of plates with various boundary conditions have been plotted. Moreover, plate's displacements under impact and moving loads have been calculated using developed SEM. The utilization of a minimum numbers of elements in SEM, consequently leading to a considerable decrease in computational costs, is the main advantage of this method.

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1. Introduction

Plates are extensively used as structural elements in manufacturing and construction of ships, space crafts, bridges, and many other structural systems. Therefore, the exploration of the dynamics response of plates is one of the most major issues and of engineers' high interest. To address modeling dynamic response of thick and thin plates, many numerical studies have been conducted and as a result, a great variety of exact and numerical methods have been developed. Finite element method (FEM) is one of the well-used methods to model dynamic behavior of structural systems. The use of FEM can lead to relatively accurate and acceptable results when the mesh size is smaller than the smallest wave length. As a guidance, mesh size must be 10–20 times smaller than the smallest wave length (corresponding to the greatest frequency) in order to obtain reasonably accurate results [1]. For dynamic problems, involving high frequencies, the number of mesh increases significantly, and it leads to high computational demand.

Time domain polynomial functions used in FEM cannot provide all essential frequencies. If frequency shape functions of a structure are accessible, there is no need to apply mesh; moreover, access to all essential frequencies is possible, known as dynamic stiffness matrix (DSM) method. The DSM is produced in frequency domain and through an exact solution of

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http://dx.doi.org/10.1016/j.apm.2014.11.044
0307-904X/Published by Elsevier Inc.

differential equation of motion, including stiffness, damping effects and structural mass. Because fine mesh is no longer needed, degrees of freedom (DOF) and calculation costs decrease, significantly.

A large variety of methods are used in solving the dynamic differential equations of plates in time domain. Modal analysis and solution by Fourier series are two of the most commonly used methods in category of time-domain methods. In another technique, the differential equation of plate can be solved through superposition of an unlimited number of wave modes, including all frequencies (frequency domain methods). In this technique, called as Spectral Analysis Method (SAM), Fourier coefficients are calculated in frequency domain and the response in time domain is gained by means of Inverse Fourier Transform (IFT). Where the load function is simple, the use of continuous Fourier Transform (FT) is a decent act, but in most scientific problems with complex or experimental loading, Discrete Fourier Transform (DFT) method is applied. FFT algorithm is used for evaluation of DFT. In this algorithm, the system response is expressed using a finite number of wave modes in a discontinuous domain. Thus, FT and IFT can be calculated easily at low computational cost. Combining DSM and SAM while using FFT algorithm brings about a method referred to as spectral element method (SEM) in literature. SEM is similar to FEM, except in SEM the dynamic stiffness matrix is formulated in every discontinuous frequency and exact shape functions are obtained through frequency-related differential equations. Minimizing the FFT and IFFT errors, it can be claim that SEM is a precise and efficient method for solving dynamic structural problems.

While many researchers have conducted many studies using SEM in recent years, Doyle is one of the first researchers to apply SEM to wave propagation in structures. In 1986, he began his work by elucidating a number of difficulties in using FFT algorithms and considering the wave propagation in a beam under an impact loading [2]. He and his colleagues have made valuable contributions for the advancement of SEM for many diverse structural elements, such as rode [3], beam [4], layered solids [5], multiple connected Timoshenko beam [6], varying cross-section beam [7], and spectral super element [8]. Their work, which is based on pertinent research at Purdue University, is summarized in [9].

In addition to Doyle and his colleagues, Lee and his colleagues also had an outstanding role in the development of SEM. Before 1996, all developed structural elements in SEM were subjected to concentrated loads. Lee and Lee developed an extended SEM to analyze a beam under dynamic distributed loading [1,10].

One of Lee's recent articles is about wave propagation in extended Timoshenko beam [11]. In addition, Lee and Jang developed the use of frequency-dependent spectral element matrix (or exact dynamic stiffness matrix) for dynamic response of composite Timoshenko beams. They obtained solutions with great accuracy while reducing the total number of degrees of freedom [12].

Combination of the Fourier spectral method and differential quadrature method in barycentric form, as a numerical method for solving problems for thin plates resting on Winkler foundations with irregular domains, was introduced by Shao and Wu [13]. In their solution, the arbitrary distributed loading was approximated by Chebyshev polynomials.

Wave propagation and transient response of an infinite functionally graded plate under a point impact load was presented by Sun and Lou [14]. In their work, the analytic dispersion relation of the functionally graded plate was obtained by integral transforms, and a complete discussion of dispersion was provided on the functionally graded plate.

Zhou and Wong extracted the formulation for natural frequency of thin circular and annular plates using the Hamiltonian approach [15]. The separation of variables technique was employed to solve Hamiltonian dual equations of eigenvalue problem. Analytical frequency equations were obtained based on different cases of boundary conditions.

Park and Lee used SEM to develop dynamic formulation of composite beams in the frequency domain. In their paper, the axial-bending coupled equations of motion and boundary conditions were derived for two-layer smart composite beams using the Hamilton principle with Lagrange multipliers [16].

In this paper, the spectral dynamic stiffness matrix for Levy-type thick plate is developed and the derived dynamic stiffness matrix is used to extract natural frequencies and displacements of the plate with uniform and tapered thickness subjected to impact and moving loads. Efficiency of the developed method in comparison to other numerical methods has been evaluated using the least number of elements to solve any desired problem.

2. Equation of motion and spectral element formulation

A moderately thick rectangular Levy-type plate subjected to the external dynamic load $f(x,y,t)$ with two simply supported edges parallel to the y-axis and two edges with arbitrary boundary conditions parallel to the x-axis is considered. As shown in Fig. 1, plate has uniform thickness h and dimensions l and a in x and y directions, respectively. The displacement fields, based on the first order shear deformation plate theory (Mindlin's plate theory) can be expressed in the following form:

$$u = z\psi_x(x,y,t), \quad v = z\psi_y(x,y,t) \quad w = w(x,y,t), \tag{1}$$

where, t is the time; u, v and w are the corresponding displacement components of a point in mid-surface of the plate in x, y and z directions, respectively. ψ_x and ψ_y are the rotation of mid-plane in x–z (about y-axis) and y–z plane (about x-axis), respectively.

Using Hamilton's principle and Eqs. (1), the governing equations of motion for moderately thick plate are derived as follows [17]:

$$\kappa^2 Gh\left[\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x} - \psi_x\right) + \frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y} - \psi_y\right)\right] + f(x,y,t) = \rho h\frac{\partial^2 w}{\partial t^2} + c_0\frac{\partial w}{\partial t},$$