Applied Mathematical Modelling 38 (2014) 451–463

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Analytical solutions for movement of cold water thermal front in a heterogeneous geothermal reservoir

Sayantan Ganguly [a], M.S. Mohan Kumar [b,*]

[a] Department of Civil Engineering, Indian Institute of Science, Bangalore 560 012, India
[b] Department of Civil Engineering, Indo-French Cell for Water Sciences, Indian Institute of Science, Bangalore 560 012, India

ARTICLE INFO

Article history:
Received 5 July 2012
Received in revised form 15 March 2013
Accepted 24 June 2013
Available online 5 July 2013

Keywords:
Geothermal reservoirs
Analytical solutions
Reinjection
Thermal front
Heterogeneous porous media
Heat transport

ABSTRACT

In the present study an analytical model has been presented to describe the transient temperature distribution and advancement of the thermal front generated due to the reinjection of heat depleted water in a heterogeneous geothermal reservoir. One dimensional heat transport equation in porous media with advection and longitudinal heat conduction has been solved analytically using Laplace transform technique in a semi infinite medium. The heterogeneity of the porous medium is expressed by the spatial variation of the flow velocity and the longitudinal effective thermal conductivity of the medium. A simpler solution is also derived afterwards neglecting the longitudinal conduction depending on the situation where the contribution to the transient heat transport phenomenon in the porous media is negligible. Solution for a homogeneous aquifer with constant values of the rock and fluid parameters is also derived with an aim to compare the results with that of the heterogeneous one. The effect of some of the parameters involved, on the transient heat transport phenomenon is assessed by observing the variation of the results with different magnitudes of those parameters. Results prove the heterogeneity of the medium, the flow velocity and the longitudinal conductivity to have great influence and porosity to have negligible effect on the transient temperature distribution.

1. Introduction

In a geothermal power plant reinjection of the heat depleted water extracted from the geothermal reservoir has been a common practice for quite some time. This started for safe wastewater disposal [1] and later on the technology was employed to obtain higher efficiency of heat and energy extraction [2]. According to Bodvarsson and Stefansson [3] in most of the cases a very small fraction of the thermal energy present in the reservoir can be recovered without the reinjection of geothermal fluid. Also form the study of Bodvarsson [4] maintaining the reservoir pressure is essential which gradually reduces due to continuous extraction of reservoir fluid without reinjection especially for reservoirs with low permeabilities. Although reinjection of cold water has several benefits, the possibility of premature breakthrough of the cold water front, from injection well zone to production well zone, reduces the efficiency of the reservoir operation drastically. Hence for maintaining the reservoir efficiency and longer life of the reservoir, the injection-production well scheme is to be properly designed and injection and extraction rates are to be properly fixed.

The phenomenon of advancement of thermal front in porous and fractured media has been studied analytically, numerically and experimentally by a number of researchers. Lauwerier [5] developed a mathematical model using two-dimensional Laplace transform to analyze the movement of the thermal front generated due to the injection of hot

* Corresponding author. Tel.: +91 80 22932814/23600910; fax: +91 80 23600404.
E-mail addresses: sayantan@civil.iisc.ernet.in (S. Ganguly), msmk@civil.iisc.ernet.in (M.S. Mohan Kumar).

0307-904X/$ - see front matter © 2013 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.apm.2013.06.031

water into an oil reservoir. Bodvarsson [4] derived analytical models of temperature distribution for flow through a single fracture and laminated solids, i.e. solids consisting of impermeable rock with equidistant permeable fractures. The analytical solutions developed by him include constant mass flow input with time-varying temperature and weakly oscillating mass flow. Bodvarsson [6] presented a simple mathematical model for transient temperature field in the reinjection zone for flow through both porous and fractured media. The paper also discusses about some practical issues related with the siting of reinjection wells. The reservoir lifetime and energy extraction efficiency depends a lot on reinjection of heat depleted water. The heat recovery factor and aquifer production potential is enhanced highly by reinjection process Gringarten [2].

Bodvarsson and Tsang [7] also presented another analytical model to investigate the movement of the cold water front through equally spaced horizontal fractured system with impermeable rock separating them. Numerical modeling was also performed by them to judge the importance of the assumptions applied in analytical modeling and to perform the same study with permeable rock matrix. Their results have shown that the assumption of negligible longitudinal conductive heat transport leads to erroneous temperature distribution at very large times. Garg and Pritchett [8] developed an approximate analytical solution to solve the pressure build up in a geothermal reservoir (both single and two phase) due to cold water injection. They also derived approximate solutions for the pressure fall response after the injection was stopped. Numerical simulation was also performed by them to verify the pressure transient analysis obtained from the analytical model. Stopa and Wojnarowski [9] examined the propagation of the cold water thermal front by developing an analytical model where they considered heat capacity, density of rock and water to be functions of the temperature. An experimental study was done by Li et al. [10] to investigate the effects of temperature and pressure on the *in situ* saturation of the geothermal reservoir. They also examined the effect of reservoir pressure on reservoir productivity.

In spite of having a handful of analytical models in this field, none of them consider the geothermal reservoir media to be heterogeneous or the variation of flow velocity and other thermal properties of rock and fluid to be functions of space. The assumption of homogeneous aquifer is an idealistic one. Practically there is always some heterogeneity in a porous aquifer and hence inclusion of that in a model is also crucial. The main objective of the present study is to investigate the transient temperature distribution in a heterogeneous geothermal medium or the advancement of the cold thermal front generated due to injection of the heat depleted water into the hot geothermal reservoir medium. Complete and exact analytical solutions are derived using Laplace transform technique wherein the quasilinear partial differential equations are transformed to constant coefficient differential equations applying a variable transformation technique [11]. The heat transfer processes taken into account are one-dimensional advection and longitudinal conduction through the porous geothermal aquifer. The study is needed for the design of the injection-production well scheme and fixing the injection rate such that the advancement of the cold water front can be controlled in order to maintain the reservoir efficiency for a longer period of time. The results derived can also serve as a reference solution for the complex numerical models for more realistic practical problems.

2. Mathematical formulation

One-dimensional heat transport equation in porous medium is considered here to describe the transient temperature distribution in the geothermal aquifer. Cold water is injected into the geothermal aquifer situated at one end of the aquifer domain (Fig. 1). The porous medium is considered to be heterogeneous and it introduces a variation of flow velocity in the medium. The thermal conductivity of the aquifer porous medium which varies spatially due to heterogeneity of the medium is also considered. The assumptions which are applied to derive the analytical solution are given by

1. The geothermal aquifer is a heterogeneous one. The heterogeneity consequences in spatial variation of velocity which is considered to be linearly increasing with distance [11,12] in a finite domain $0 \leqslant x \leqslant L$ and is given by

$$u_w(x) = u_0(1 + ax), \tag{1}$$

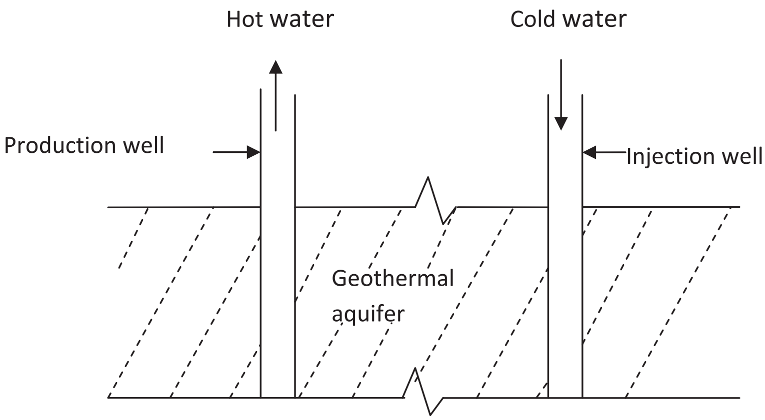

Fig. 1. Schematic diagram of the geothermal aquifer with production and injection well installed.