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Elastic stability of all edges simply supported, stepped and stiffened rectangular plate under Biaxial loading

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ABSTRACT

In this paper using finite difference method the lower bound buckling load for simply supported (a) stepped and stiffened rectangular thin plate (b) linear and non-linear variation of thickness (c) uniformly distributed compressive forces in both directions (d) uniformly distributed compressive force in y direction and non-uniform distribution of compressive force in x-direction is discussed. The thin plate is divided into 900 rectangular meshes. The partial derivatives are approximated using central difference formula. Eight hundred and forty one equations are formed and using the program developed and the least eigenvalue is obtained. The buckling coefficients are calculated for different types of stepped and non prismatic plates and the results are presented in tables and graphs for ready use by designers. Buckling factors for some cases are presented in the form of three separate tables and compared with the values obtained by Xiang, Wei and Wang. The results are in close agreement.

1. Introduction

Plates with varying thickness are extensively used in modern structures due to their unique functions. For example stepped plates possess a number of attractive features, such as material saving, weight reduction, stiffness enhancing, designated strengthening etc. In particular, buckling of stepped plates has attracted much attention in the past few decades. This type of plate is used extensively because of its high strength to weight ratio. A variety of theoretical, approaches have been formulated for this class of problems. The buckling loads for uniform thickness plates have been summarized in the text by Timoshenko and Gere [1], by Allen and Bulson [2] and also by Szilard [3]. Wittrick and Ellen [4] solved the problem of plates tapering in one direction and uniformly compressed in that direction by using Galerkin's method. Manisfield [5] has done research on the buckling of certain optimum plate structures with linearly varying thickness. He solved the general differential equation with varying flexural rigidity. Perturbations technique was applied by Chehil and Dua [6] to solve the stability problems of a plate.

Hwang [7] investigated the stability of plates with piecewise variation in thickness using energy method by applying the principle of virtual displacement. The implementation of energy method in stability of plates was explained by Chen and Lui [8]. Singh and Dey [9] applied variational finite difference approach for bi-directionally stepped plates. Harik et al. [10] used a semi numerical–semi analytical method for the analysis of plates with varying rigidities. They first reduced the governing partial differential equation to an ordinary differential equation using the method of separation of variables, and then finite difference technique was used to solve the ordinary differential equation. Subramanian et al. [11] analyzed elastic stability of

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Nomenclature

a	length of the plate
a_1	step length with thickness t_0
b	width of the plate
a/b	panel aspect ratio
C_l	critical load
D	flexural rigidity of the plate
E	Young's modulus
$\mathbf{h_0}$	geometric stiffness matrix
k	buckling coefficients
k_1	buckling coefficient (finite difference method)
k_2	buckling coefficient (energy method)
$[K]$	coefficient matrix
$[K_G]$	coefficient matrix
l	mesh length
m	mesh breadth
$\mathbf{M_0}$	flexural stiffness matrix
M	the number of divisions in the x-direction
N	the number of divisions in the y-direction
N_x, N_y, N_{xy}	in plane forces
nj	number of joints
t	thickness of the plate
t_i	step thickness
$\underline{\mathbf{w}}$	displacement vector
w	lateral deflection
(x, y)	Cartesian coordinates
α	ratio between the breadth and length of a mesh
ν	Poisson's ratio
λ	eigenvalue

varying thickness plates using the finite element method with uniformly distributed compressive forces in one direction. Bradford and Azhari [12] used finite strip method using two types of series functions to find the elastic local buckling of plates with different boundary conditions. Nerantzaki and Katsikadelis [13] solved for the buckling load of simply supported plate with linear and exponential variation in loaded direction by using the analog equation method. Xiang and Wang [14] presented an analytical approach that combines Levy method and the state space technique for determining exact buckling and vibration solutions of uni-directional multi-stepped rectangular plates. The extended Kantorovich method in conjunction with the exact element method was used by Eisenberger and Alexander [15]. Exact solutions for buckling and vibration of stepped rectangular Mindlin plates were obtained by Xiang and Wei [16] by using Levy's solution and a decomposition method. John Wilson and Rajasekaran [17] used finite difference technique to determine the buckling load for a stepped plate with all edges simply supported under uni-axial loading.

In this paper a finite difference method is used to solve the fourth order partial differential equation for stepped and stiffened simply supported plates with biaxial compressed load. The stability load of a plate always can be increased by increasing its thickness, but such a design will not be economical with respect to the weight of the material used. A more economical solution is obtained by keeping the thickness of the plate as small as possible and increasing the stability by introducing reinforcing ribs. For different panel and thickness ratios, the critical values are calculated for single, double stepped rectangular plates and also for longitudinal, latitudinal stiffened plates.

2. Formulation of the problem

Consider a rectangular plate of side 'a' and 'b' as shown in Fig. 1. Divide the plate into $M \times N$ rectangular meshes of length such that $l = a/M; m = b/N$. To solve the problem by finite difference approach one has to assume imaginary points as shown in Fig. 2.The total number of unknowns will be the deflections at all the $nj = MN + 3M + 3N - 3$ joints. (Total number of joints = internal points + boundary points + imaginary points).

Boundary equations available are

1. Equilibrium equations to be provided at all the internal points $neq = (M - 1)(N - 1)$ as