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Spatial distribution complexities of traffic congestion and bottlenecks in different network topologies

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ABSTRACT

Recently, urban traffic congestion has become a popular social problem. The generation and the propagation of congestion has close relation with the network topology, the traffic flow, etc. In this study, based on the traffic flow propagation method, we investigate the time and space distribution characteristics of the traffic congestion and bottlenecks in different network topologies (e.g., small world, random and regular network). The simulation results show that the random network is an optimal traffic structure, in which the traffic congestion is smaller than others. Moreover, the regular network is the worst topology which is prone to be congested. Additionally, we also prove the effects of network with community structure on the traffic system and congestion bottlenecks including its generation, propagation and time–space complexities. Results indicate that the strong community structure can improve the network performance and is effective to resist the propagation of the traffic congestion.

1. Introduction

Urban traffic system can be regarded as the combination of different elements and their interactions, which includes three sub-systems: road, flow and management. Uncoordinated behaviors among three systems would induce the traffic problems, such as congestion bottlenecks. A bottleneck is a spatial discontinuity where road capacity is reduced, is the original sources of the grievous traffic problem and has become a key factor in counteracting traffic to be free. A bottleneck will frequently deduce the congestion emergences and queues formation, which will consequently aggravate travel delay, induce traffic congestion, and worsen urban traffic environment and safety. In fact, the effects of a bottleneck on the traffic flow have been studied in the "Lighthill, Whitham and Richards" (LWR) framework [1,2]. Following this work, some approaches are proposed to analyze the traffic flow in the bottleneck with different methods. The entropic solution of the LWR model on both sides of a fixed bottleneck was established by [3,4] and more by Jin and Zhang [5] in a more different way. Newell [6] developed a queuing model at freeway bottlenecks. Daganzo and Laval [7] and Ni and Leonard [8] proposed two models to capture the flow characteristics of the merge bottleneck. Some works focus on the analysis of the congestion bottleneck and the departure time choice.

The cell transmission model (CTM), as a discrete model of the LWR, was firstly presented by [9] which is used for highway traffic simulation. Then, [3] extended CTM to a network situation. As a well simulation tool, it can easily capture the realistic traffic phenomena, such as shock waves, queue formation, dissipation and queue spillback. Recently, various works had

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developed this theory. For example, Lo [21], Lo and Szeto [22] and Szeto and Lo [23] used CTM for dynamic traffic assignment (DTA) to enhance the veracity of estimating dynamic route impedance and improve the application effect of dynamic traffic model. In addition, based on the cell transmission model (CTM), Long et al. [10,11] proposed a congestion propagation model for the urban traffic and applied it to simulate the formation and dissipation of the traffic congestion.

In the previous literatures, however, the related studies mainly focus on the traffic bottleneck by simulation methods. In recent years, the study of topological and dynamical properties of traffic networks attracted much attention. Part of this interest comes from the attempt to understand the macroscopic behavior of traffic networks, i.e., topological behaviors, statistical properties, structure evolution, etc. It is proved that real cities are neither trees nor perfect grids, but a combination of these structures that emerge from the social and constructive processes [12]. Moreover, most of the urban road networks are proved to have the small world effects (a large cluster coefficient and a small average shortest path). For example, [13] concluded that the topological networks of streets in big cities exhibited homogeneous properties but were not heterogeneous networks through analysis of the topological networks in three cities. Gao et al. [14] investigated the urban road network and found a scale-free network with small-world characteristic based on the GIS technology. Lighthill and Whitham [15] studied the transit network of Beijing and found its small world structure.

In order to identify the bottleneck in the traffic network, the propagation of traffic congestion can be considered. Since the dynamic propagation of the traffic flow is extremely complicated which relates to travel behaviors and network structures, it is significant to reveal the effects of network topologies on traffic system performance. Jenelius et al. [16] presented some topological measures of the road network, which can also be used as the guidance to road administrations in their prioritization of maintenance and repair of roads, as well as for avoiding causing unnecessary disturbances in the planning of roadwork. To identify the role of key components of the traffic network, [17] proposed a method in homogenous and heterogeneous topologies which provided an extended identification technology for traffic bottleneck.

However, for the dynamics traffic flow studies, they are limited to a particular network without taking the complexity of the network topology into account, and assumed that the alternative path for the traveler is unique (the shortest path). In reality, not all of the travelers can get the whole link travel information. Generally, they have some alternative path in their travel. However, for the effects of traffic topologies on the traffic system performance, most of them are based on the static user equilibrium in which the dynamic traffic flow characteristics are less considered.

In this paper, the dynamical traffic flow characteristics in different network topologies are investigated based on the link and the node propagation model proposed in our previous works [10,11] by the measures of average journey velocity, total system cost and total delay time. CTM provide the model and simulation method, while the network gives the basic topologies where CTM can be performed. The purpose of this study is to understand the complex behaviors of the traffic flow with respect to the network topology. We focus on examining the performance of the traffic system for three typical topologies, e.g., the regular, random, and small world networks. Another contribution is to explore the effects of the traffic demands on the network performance and to find the optimal network topologies. Besides, the temporal and spatial distribution of the congestion bottleneck by the total duration interval and the largest scale of the congestion bottleneck is analyzed for different topologies.

The paper will be organized as follows. Section 2 gives the traffic flow propagation model in traffic network based on CTM. Section 3 develops the performance measures of the traffic system and congestion bottleneck. The numerical examples in different network topologies, e.g., regular lattice, random graph and small world network are analyzed in Section 4. In Section 5, the distributions of congestion bottlenecks on community structure are investigated. Section 6 contains the conclusions.

2. Propagation model

2.1. Network traffic flow propagation model

Long et al. [10] established a traffic flow CTM including link and node propagation based on the fundamental work of [3]. They analyzed the congestion propagation and dissipation in two-way rectangular grid networks. We refer to [10] for a full description of the link and node propagation model.

2.1.1. The CTM link model

To simplify the solution scheme of LWR model, [9,3] proposed a famous CTM traffic flow model based on the traffic flow q and density k relationship.

$$q = \min\{vk, q_{\max}, w(k_j - k)\}, \quad 0 \leqslant k \leqslant k_j, \tag{1}$$

where k_j, $q_{\max}$, v, w denote, respectively, jam density, inflow capacity, free-flow speed and the speed of the backward shock wave (or the backward propagation speed of disturbances in congested traffic). Meanwhile, Eq. (1) approximates the fundamental diagram by a piece-wise linear model as shown in Fig. 1.

In the link model, each link a is divided into i homogeneous cells from upstream to downstream whose length is equal to the distance traveled by free-flow traffic in one time interval δ. For any link a, based on the Eq. (1) and the relationship