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A group decision making approach based on aggregating interval data into interval-valued intuitionistic fuzzy information

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ABSTRACT

Group decision making is one of the most important problems in decision making sciences. The aim of this article is to aggregate the interval data into the interval-valued intuitionistic fuzzy information for multiple attribute group decision making. In this model, the decision information is provided by decision maker, which is characterized by interval data. Based on the idea of mean and variance in statistics, we first define the concepts of satisfactory and dissatisfactory intervals of attribute vector against each alternative. Using these concepts, we develop an approach to aggregate the attribute vector into interval-valued intuitionistic fuzzy number under group decision making environment. A practical example is provided to illustrate the proposed method. To show the validity of the reported method, comparisons with other methods are also made.

1. Introduction

In [1], Zadeh introduced the concept of fuzzy set whose basic component is only a membership function with the non-membership function being one minus the membership function. However, in real-life situations, when a person is asked to express his/her preference degree to an object, it is possible that he/she is not so sure about it, that is, there usually exists a hesitation or uncertainty about the degree, and there is no means to incorporate the hesitation or uncertainty in a fuzzy set. Later, Atanassov [2] gave a generalized form of fuzzy set, called intuitionistic fuzzy set, which is characterized by a membership function and a non-membership function. Atanassov and Gargov [3] extended the intuitionistic fuzzy set to the interval-valued intuitionistic fuzzy set (IVIFS), which is characterized by a membership function and a non-membership function whose values are intervals rather than real numbers. After the pioneering work of Atanassov and Gargov [3], the IVIFS has received much attention from researchers [4–10].

Considering that, in some real-life situations, such as in an automatic air quality monitoring station, thousands of data on an index are collected everyday. How to analyze these data to find valuable changes on a concerned index is a problem which needs to be solved urgently. Whether these data can be:

(1) aggregated into some intervals, for instance, [minimum/minute, maximum/minute]; then
(2) aggregated these intervals into a special form of number, for example, interval-valued intuitionistic fuzzy number (IVIFN) [11].

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Owing to the increasing complexity in modern society, aggregating group's knowledge and experiences to make an appropriate decision is a commonly used method. This an interesting and important research topic. Until now there are a few investigations devoted to the aggregation techniques of intuitionistic fuzzy information. Yue [12] and Yue et al. [13,14] developed some approaches for the aggregation by using the Golden Section. Yue et al. [15] introduced a method for the aggregation based on the Minimax Criterion in decision method. Yue [16] proposed an approach for aggregating intervals into interval-valued intuitionistic fuzzy information for group decision making (GDM). Recently, Yue and Jia [17] described a method to aggregate crisp values into interval-valued intuitionistic fuzzy information for GDM. In this paper, we develop a new method, based on the idea of mean and variance in statistics, to aggregate interval data into interval-valued intuitionistic fuzzy information for GDM.

In order to do so, the rest of this paper is organized as follows. Section 2 reviews some basic concepts and operations related to IVIFSs. Section 3 develops a method of GDM based on aggregating interval data into interval-valued intuitionistic fuzzy information. Section 4 gives an illustrative example. Section 5 gives comparisons suggested methodology with other methods. Finally, Section 6 concludes the paper.

2. Preliminaries

Atanassov and Gargov [3] introduced the notion of IVIFS as follows.

Definition 1 [3]. Let a set $X = \{x_1, x_2, \ldots, x_n\}$ be fixed, then an IVIFS $\tilde{A}$ over X is an object having the form:

$$\tilde{A} = \{(x_i, \mu_{\tilde{A}}(x_i), \nu_{\tilde{A}}(x_i))|x_i \in X\}, \tag{1}$$

where $\mu_{\tilde{A}}(x_i) = [\mu^l_{\tilde{A}}(x_i), \mu^u_{\tilde{A}}(x_i)] \subseteq [0,1]$ and $\nu_{\tilde{A}}(x_i) = [\nu^l_{\tilde{A}}(x_i), \nu^u_{\tilde{A}}(x_i)] \subseteq [0,1]$ are intervals, $\mu^l_{\tilde{A}}(x_i) = \inf \mu_{\tilde{A}}(x_i)$, $\mu^u_{\tilde{A}}(x_i) = \sup \mu_{\tilde{A}}(x_i)$, $\nu^l_{\tilde{A}}(x_i) = \inf \nu_{\tilde{A}}(x_i)$, $\nu^u_{\tilde{A}}(x_i) = \sup \nu_{\tilde{A}}(x_i)$, and

$$\mu^u_{\tilde{A}}(x_i) + \nu^u_{\tilde{A}}(x_i) \leqslant 1, \quad \text{for all } x_i \in X \tag{2}$$

and $\pi_{\tilde{A}}(x_i) = [\pi^l_{\tilde{A}}(x_i), \pi^u_{\tilde{A}}(x_i)]$, where

$$\pi^l_{\tilde{A}}(x_i) = 1 - \mu^u_{\tilde{A}}(x_i) - \nu^u_{\tilde{A}}(x_i), \quad \pi^u_{\tilde{A}}(x_i) = 1 - \mu^l_{\tilde{A}}(x_i) - \nu^l_{\tilde{A}}(x_i), \quad \text{for all } x_i \in X. \tag{3}$$

In particular, if $\mu_{\tilde{A}}(x_i) = \mu^l_{\tilde{A}}(x_i) = \mu^u_{\tilde{A}}(x_i)$ and $\nu_{\tilde{A}}(x_i) = \nu^l_{\tilde{A}}(x_i) = \nu^u_{\tilde{A}}(x_i)$, then, $\tilde{A}$ is reduced to an intuitionistic fuzzy set [2].

The IVIFS has a physical interpretation [17] as follows.

Example 1. In a decision making problem, let $A = \{A_1, A_2, \ldots, A_m\}$ be a set of alternatives, $U = \{u_1, u_2, \ldots, u_n\}$ be a set of attributes. If we consider U as a universe of discourse, and each alternative A_i is evaluated by using IVIFS, then the alternative score can be represented as

$$\tilde{A_i} = \{< u_j, ([\mu^l_{\tilde{A}_i}(u_j), \mu^u_{\tilde{A}_i}(u_j)], [\nu^l_{\tilde{A}_i}(u_j), \nu^u_{\tilde{A}_i}(u_j)]) > |u_j \in U\}, \quad i = 1, 2, \ldots, m,$$

where $[\mu^l_{\tilde{A}_i}(u_j), \mu^u_{\tilde{A}_i}(u_j)]$ indicates the degree that the alternative A_i satisfies the attribute u_j, and $[\nu^l_{\tilde{A}_i}(u_j), \nu^u_{\tilde{A}_i}(u_j)]$ indicates the degree that the alternative A_i does not satisfy the attribute u_j, with the condition $\mu^u_{\tilde{A}_i}(u_j) + \nu^u_{\tilde{A}_i}(u_j) \leqslant 1$. For example,

$$\tilde{A_1} = \{\langle u_1, ([0.4, 0.6], [0.2, 0.3])\rangle, \langle u_2, ([0.2, 0.5]), [0.3, 0.4])\rangle\},$$

then it can be interpreted as "if 10 experts take part in the evaluation of alternative A_1 with respect to the attribute set $U = \{u_1, u_2\}$, the votes for resolution on u_1 are 4–6 in favor, 2–3 against, and 1–4 abstentions, where $\pi^l_{\tilde{A}_1}(u_1) = 1 - \mu^u_{\tilde{A}_1}(u_1) - \nu^u_{\tilde{A}_1}(u_1) = 1 - 0.6 - 0.3 = 0.1$, $\pi^u_{\tilde{A}_1}(u_1) = 1 - \mu^l_{\tilde{A}_1}(u_1) - \nu^l_{\tilde{A}_1}(u_1) = 1 - 0.4 - 0.2 = 0.4$; the votes for resolution on u_2 are 2–5 in favor, 3–4 against, and 1–5 abstentions, where $\pi^l_{\tilde{A}_1}(u_2) = 1 - \mu^u_{\tilde{A}_1}(u_2) - \nu^u_{\tilde{A}_1}(u_2) = 1 - 0.5 - 0.4 = 0.1$, $\pi^u_{\tilde{A}_1}(u_2) = 1 - \mu^l_{\tilde{A}_1}(u_2) - \nu^l_{\tilde{A}_1}(u_2) = 1 - 0.2 - 0.3 = 0.5$.

Xu and Chen [11] called an ordered pair $\tilde{\alpha} = (\mu_{\tilde{\alpha}}(x_i), \nu_{\tilde{\alpha}}(x_i))$ an IVIFN, and denoted an IVIFN by $([a,b],[c,d])$, where

$$[a,b] \subseteq [0,1], \quad [c,d] \subseteq [0,1], \quad b + d \leqslant 1. \tag{4}$$

The IVIFN has a physical interpretation [17] as follows.

Example 2. As mentioned in Example 1, if alternative A_i is evaluated by using IVIFS, then each attribute (u_j) score is an IVIFN, which can be written as $([a_{ij}, b_{ij}], [c_{ij}, d_{ij}])$, where $[a_{ij}, b_{ij}]$ indicates the satisfactory interval that the alternative A_i satisfies the attribute u_j, and $[c_{ij}, d_{ij}]$ indicates the dissatisfactory interval that the alternative A_i dose not satisfy the attribute u_j.

Let $\tilde{\alpha}_1 = ([a_1, b_1], [c_1, d_1])$, $\tilde{\alpha}_2 = ([a_2, b_2], [c_2, d_2])$ and $\tilde{\alpha} = ([a,b],[c,d])$ be three IVIFNs, then [18].

(1) $\tilde{\alpha}_1 + \tilde{\alpha}_2 = ([a_1 + a_2 - a_1a_2, b_1 + b_2 - b_1b_2], [c_1c_2, d_1d_2])$;
(2) $\lambda\tilde{\alpha} = ([1 - (1-a)^\lambda, 1 - (1-b)^\lambda], [c^\lambda, d^\lambda])$, $\lambda > 0$.