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Dynamic pricing inventory control under fixed cost and lost sales

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ABSTRACT

This paper studies a periodic review pricing and inventory replenishment problem which encounters stochastic demands in multiple periods. In many inventory control problems, the unsatisfied demand is traditionally assumed to be backlogged but in this paper is assumed to be lost. In many practical problems, a consumer who could not buy what he/she wants in one store is not willing to wait until that store restocks it but tries to buy alternatives in other stores. Also, in this paper, the random variable for the demand function is assumed to be general, which means that any probability function for the random variable can be applied to our result. Cost terms consist of the holding cost by the leftover, the shortage cost by lost sales, and the strictly positive fixed ordering cost. The objective of this paper is to dynamically and simultaneously decide the optimal selling price and replenishment in each period by maximizing the expected profit over the finite selling horizon. We show that, under the general assumption on the random variable for the demand, the objective function is K-concave, an (s, S) policy is optimal for the replenishment and the optimal price is determined based on the inventory level after the replenishment in each period.

1. Introduction

In this paper, we consider a periodic review pricing and inventory replenishment problem which encounters stochastic demand under a situation with fixed ordering cost and lost sales. In many literatures considering inventory control problem, the unsatisfied demand is assumed to be backlogged. However, we can easily observe that any demand unsatisfied by one seller is lost and thus is satisfied by others. So, in practical settings, it is often reasonable that the unsatisfied demand is assumed to be lost with some cost. Our objective is to decide the optimal dynamic pricing and inventory control policy that maximizes the seller's the expected profit over the finite selling horizon.

Dynamic pricing and inventory control in the existing operations management literature can be classified based on whether the unsatisfied demand in one period is backordered or lost under the assumption of demand function. The optimality of the (s, S, p) policy has been showed for backlogging by [1] and lost-sales models by [2]. Although backordering assumption in [1] could be realistic in some manufacturing environment, it is quite limitary in a retail environment. However, even though [2] shows the optimal policy under the assumption of lost sales, they assume that the demand is a continuous random variable, distributed over the bound $[L, U]$ with a known probability density function, where L and U were differentiable. Also, [3,4] show that (s, S) policy is optimal and the value of the optimal price depends on the inventory level after the replenishment decision has been done under the assumption of lost sales. However, they assume that demand is a

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continuous random variable with increasing failure rate. The assumption on the demand function is quite restrictive. Another related research is [5], in which a dynamic pricing problem during a finite selling horizon is considered. However, they consider only one replenishment opportunity in the first period, in comparison with a dynamic inventory control assumption in this research.

However, in many practical settings, the demand is not continuous random variable, does not have a probability density function, does not have a property of increasing failure rate or the bounds are not necessarily differentiable. For example, the empirical distribution function using the historical demand data is widely used to forecast the demand, which is not continuous and moreover does not have a probability density function. So, to the best of our knowledge, the structure of the optimal policy for a lost-sales under the general random variable for demand including continuous and discrete random variable without probably density function has not been analyzed so far.

2. Notations and assumptions

Notations:

1. $K \equiv$ fixed ordering cost.
2. $c \equiv$ unit purchasing/production cost.
3. $h \equiv$ unit holding cost.
4. $b \equiv$ unit shortage cost.
5. $p_t \equiv$ unit selling price on $[\underline{p}_t, \overline{p}_t]$ in period t.
6. $\epsilon_t \equiv$ random variable for demand in period t.
7. $D_t(p_t) + \epsilon_t \equiv$ demand function in period t.
8. $x_t \equiv$ inventory level before replenishment at the beginning of period t.
9. $y_t \equiv$ inventory level after replenishment at the beginning of period t.

Assumption 1. Replenishment becomes available instantaneously.

Assumption 2. Any unsatisfied demand is lost with a shortage cost per unit of unsatisfied demand.

Assumption 3. For all t = 1,2,. . .,T, the demand function over $[\underline{p}_t, \overline{p}_t]$ is given by

$$D_t(p_t) + \epsilon_t,$$

where ϵ_t can be any random variable with $E[\epsilon_t] = 0$. Furthermore, the inverse of D_t, denoted by D_t^{-1}, is continuous and strictly decreasing. The revenue is given by

$$p_t(D_t(p_t) + \epsilon_t).$$

By Assumption 3, we can express the revenue using D_t^{-1} and it is given by

$$D_t^{-1}(d_t)(d_t - \epsilon_t),$$

where $d_t = E[D_t(p_t) + \epsilon_t] = D_t(p_t)$ and d_t is defined over $[D_t^{-1}(\overline{p}_t), D_t^{-1}(\underline{p}_t)]$. Thus, the expected revenue using D_t^{-1} is given by

$$E\left[D_t^{-1}(d_t)(d_t - \epsilon_t)\right] = d_t D_t^{-1}(d_t).$$

Assumption 4. For all t = 1,2,. . .,T, the expected revenue given by

$$d_t D_t^{-1}(d_t)$$

is concave in $d_t = E[D_t(p_t) + \epsilon_t] = D_t(p_t)$

3. Mathematical modeling

In this section, a mathematical model maximizing expected profit is introduced. The planning horizon consists of finite T periods, numbered $1, 2, \ldots, T-1, T$. During the planning horizon, the price can be dynamically increased or decreased. Since the price can be changed over periods, the problem can be formulated as a dynamic programming model with x_t as the state of the system at the beginning of period t. Let $\Pi_t(x_t)$ be the maximum expected profit from t to T. Thus, in period t, the maximum expected profit is given by

$$\Pi_t(x) = cx + \max_{y \geqslant x} - K1_{\{y>x\}} + G_t(y, d(y)),$$