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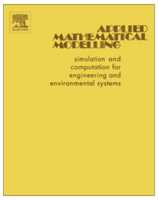

Finite volume analysis of adaptive beams with piezoelectric sensors and actuators

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ABSTRACT

This paper presents a finite volume (FV) formulation for the free vibration analysis and active vibration control of the smart beams with piezoelectric sensors and actuators. The governing equations based on Timoshenko beam theory are discretized using the finite volume method. For the purpose of forced vibration control of beam structures, the negative velocity feedback controller is designed for the single-input, single-output system. To achieve the best effect, the piezoelectric sensors and actuators are coupled with the host structure in different positions and then the performance of the designed control system is evaluated for each position. In the test examples, first the shear locking free feature of the present formulation is demonstrated. This has been performed by doing static and natural frequency analysis of some reference models. Then, the capability of the proposed method for the prediction of uncontrolled forced vibration response and active vibration control of a beam structure is studied.

1. Introduction

Smart structure has been an interesting topic of research since the last two decades. This intelligent system composed of an instructing unit incorporated with smart materials such as piezoelectric, shape memory alloy, electro rheological and magneto rheological fluids and so on acting as sensor and actuator on the host structure. The intelligence and accountability of aforementioned system in dealing with structural vibration has made it attractive in many engineering applications. However, utilizing smart materials in civil engineering applications has some limitations and few researches concerning active vibration control of building structures by smart materials have been published so far. However, due to some unique features of smart materials such as fast response to the input signals, adaptability, cheapness, and lightness, a promising future of their civil engineering applications can be expected.

Piezoelectric is one of the most popular smart materials which are widely used in vibration control of smart structures. Inherent reciprocal effect of electrical and mechanical domains, which is called electromechanical coupling, provides a feasible use of piezoelectric materials as both sensors and actuators. In sensor patches, the mechanical variables are exchanged into the electrical variables and vice versa, in actuators the electrical variables are exchanged into the mechanical variables.

There have been many researches on the subject of piezoelectric sensors and actuators applications [1–3]. These works rather focused on engineering performances of the piezoelectric sensors and actuators in structural vibration control and few studies were carried out on the accuracy and efficiency of the numerical procedures used for the modelling.

Finite elements method (FEM) is the most powerful numerical technique to simulate the behaviour of piezoelectric smart structures. Several aspects of piezoelectric smart structures modelling using FEM can be found in many research works.

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Vibration control of composite beam by taking into consideration of first-order shear deformation theory [4], study of shear-based piezoelectric actuation in vibration control of sandwich beams [5], demonstration of FE modelling accuracy of an active cantilever plate behaviour using an experimental model [6] and finite element model for analysis and design of active vibration control of smart plates [7,8] are examples of these studies. Besides, other numerical techniques such as boundary element method (BEM) and differential quadrature method (DQM) were also utilized for modelling of smart structures [9,10]. Although the above mentioned approaches give proper results, comparison of different numerical methods ability in simulating of complex composite structures and introducing a superior technique which predicts the structural behaviour more factual, seems to be necessary.

Finite volume method (FVM), which is a traditional method in fluid mechanics, has been employed for the analysis of solid mechanics problems in the last two decades. Simplicity in concept and formulation, shear locking free feature in the analysis of thin Timoshenko beam model and in the bending analysis of thin Mindlin plates and also having the capability in accurate prediction of structural responses, are the most distinguished characteristics of this procedure [11]. Two common schemes for FVM have been reported in literature; the cell-vertex approach and the cell centred one [12,13]. In the cell-vertex approach, the field variables are computed at vertices of cells. For this purpose a control volume is constructed for each mesh vertex by joining the centre of surrounding cells to the middle of its adjacent sides; whereas in the cell-centred method each element acts as a control volume or cell and field variables are associated with the cell centres. In recent years, the FV methods have been applied to a number of solid mechanics problems. The solutions of solid mechanics problems with material non-linearity [14,15], FV analysis of dynamic fracture phenomena [16–18], analysis of two-and three-dimensional general structural dynamic problems [19,20], stress analysis of geometrically nonlinear solids [21,22], bending analysis of plates [11,23–25] and FV formulation for fluid-structure interaction [26,27] are among the literature concerning on finite volume procedure.

The finite volume method has been also extended to model materials with heterogeneous microstructures, including periodic and functionally graded materials [28–32] and also applied to the elastic–plastic analysis of these types of materials [33].

The capability and efficiency of FV method in computational solid mechanics has been examined in the above works. In authors' knowledge there is no published work concerning the finite volume application for the modelling of smart structures. However, the ability of more accurate structural response prediction by FV can lead to more efficient control of vibrations of smart structures, benefitted by a feedback controller.

This paper addresses the finite volume formulation for static and dynamic behaviour of composite beams. At first, the efficiency of FVM in terms of accuracy and non-locking behaviour is demonstrated in some benchmark problems such as the cantilever and simply supported beams. Thereafter, by incorporating the smart layers acting as sensor and actuator into the main structure and using a feedback controller, the capability of present formulation in numerical simulation of active vibration control of smart beams is analysed. The effects of smart layers' locations for the efficient vibration suppression of beam are studied. For this purpose, four different scenarios of smart layers arrangements are considered and optimal arrangement is determined.

2. Basic formulation

2.1. Piezoelectric constitutive equations

The constitutive equations of a piezoelectric material, describing the electromechanical coupling are given as

$$\begin{aligned} \boldsymbol{\sigma} &= \boldsymbol{C}\boldsymbol{\varepsilon} - \boldsymbol{e}\boldsymbol{E}, \\ \boldsymbol{D} &= \boldsymbol{e}^T\boldsymbol{\varepsilon} + \boldsymbol{\kappa}\boldsymbol{E}, \end{aligned} \tag{1}$$

where $\boldsymbol{\sigma}$, $\boldsymbol{\varepsilon}$, $\boldsymbol{D}$ and $\boldsymbol{E}$ are the elastic stress, elastic strain, electric displacement and electric field vectors, respectively, $\boldsymbol{C}$ is the elasticity matrix, $\boldsymbol{e}$ is the piezoelectric matrix and $\boldsymbol{\kappa}$ is the dielectric permittivity matrix. The electrical field is the negative gradient of the electrical potential ϕ, which is applied in the thickness direction.

The first above equation is used to determine the stresses resulting from applied voltage to the piezoelectric actuators, while the second one presents the accumulated charges on the unit area of electrodes of sensors created by the structural strains.

2.2. Displacement formulation

In the present study, the first order shear deformation theory (FSDT) or Timoshenko model is used to model a layered beam composed of a core and two outer layers of piezoelectric material. In this model the displacement fields are given as

$$\begin{aligned} u(x,z,t) &= z\theta(x,t), \\ v(x,z,t) &= 0, \\ w(x,z,t) &= w(x,t), \end{aligned} \tag{2a}$$