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An interval uncertain optimization method for vehicle suspensions using Chebyshev metamodels ☆

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ABSTRACT

This paper proposes a new design optimization framework for suspension systems considering the kinematic characteristics, such as the camber angle, caster angle, kingpin inclination angle, and toe angle in the presence of uncertainties. The coordinates of rear inner hardpoints of upper control arm and lower control arm of double wishbone suspension are considered as the design variables, as well as the uncertain parameters. In this way, the actual values of the design variables will vary surrounding their nominal values. The variations result in uncertainties that are described as interval variables with lower and upper bounds. The kinematic model of the suspension is developed in software ADAMS. A high-order response surface model using the zeros of Chebyshev polynomials as sampling points is established, termed as Chebyshev metamodel, to approximate the kinematic model. The Chebyshev meta-model is expected to provide higher approximation accuracy. Interval uncertain optimization problems usually involve a nested computationally expensive double-loop optimization process, in which the inner loop optimization is to calculate the bounds of the interval design functions, while the outer loop is to search the optimum for the deterministic optimization problem. To reduce the computational cost, the interval arithmetic is introduced in the inner loop to improve computational efficiency without compromising numerical accuracy. The numerical results show the effectiveness of the proposed design method.

1. Introduction

The kinematic characteristics of vehicle suspension systems have main effect on vehicles' handling performance. Since the positions of hardpoints of the suspension system will largely determine its kinematic characteristics, the coordinates of these hardpoints are commonly considered as design variables to optimize the suspension system performance. Due to the complex of suspension system, the models are usually developed using some commercial software, which may reach a comparable level of accuracy in engineering. However, the computation is expensive for complex models in optimization. So the approximation or metamodeling methods are often used in engineering optimization to save the computational cost [1] for real-world problems.

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Building a metamodel normally involves two steps: (1) employing design of experiments to sample the computer simulation, and (2) selecting an approximation model to represent the data and fit the model with the sample data [2]. Over the past, a number of metamodels have been developed, such as the polynomial regression model [3], Kriging model [2], radial basis functions (RBF) [4–6], and multivariate adaptive regression splines (MARS) [7]. Jin et al. [8] studied four popular metamodeling techniques, including the polynomial regression, Kriging model, MARS, and RBF based on multiple performance criteria. The results showed that the polynomials regression model has advantages such as efficiency, transparency, and conceptual simplicity over the other metamodels. Although the approximation accuracy of traditional quadratic polynomial models is not good enough for strong non-linear problems, it can be improved by using higher order polynomial model. Therefore, the high order polynomial will be used in this paper.

As aforementioned, as the first step in the construction of a metamodel, the choice of sampling points is of great importance. The larger the number of sampling points, the higher the possibility to describe the unknown process, but also the higher computational cost [9]. Thus, one of the issues of the sampling methods is how to obtain unknown information in terms of a limited number of sampling points with reasonable computational cost and numerical accuracy. One of the traditional sampling methods is the Design of Experiments (DOEs), which includes a family of methods, such as the Factorial Design (FD) [3], Central Composite Design (CCD) [10], Pseudo-Monte Carlo Sampling (PMCS) [11], and Quasi-Monte Carlo Sampling (QMCS) [11–13]. Amongst these methods, FD and CCD belonging to the classical experimental designs [9] have been widely used to construct polynomials models. The level of FD and CCD are usually selected as uniform grid in the whole design space, which may not be optimal. Thus, we use the zeros of Chebyshev polynomials [14] as the level of FD to implement a new experimental test design, to obtain higher accuracy.

So far the majority of works in vehicle dynamics are based on the assumption that all parameters of vehicle systems are deterministic. However, a number of real-world problems are too complex to be defined deterministically due to the lack of the sufficient information. Actually the uncertainties are inherent in loads, parameters, material properties, fraction tolerance, boundary conditions and geometric dimensions [15] in the whole life cycle of design, manufacturing, service and aging. The deterministic assumption may lead to designs which cannot satisfy the expected performance goal or even unfeasible designs. Hence, there is an increasing demand to consider the impact of uncertainties quantitatively in the optimization of vehicle systems due to unavoidable variability and uncertainty, in order to enhance vehicle performance and safety. The Reliable-Based Design Optimization (RBDO) and the Robust Design Optimization (RDO) represent two major paradigms for the design optimization under uncertainty [16]. RBDO methods are characterized by the use of analytical techniques to find a particular point in the design space, which is related to the probability of the system failure, defined by a limit state function. This point is often referred to the most probable point (MPP) or the design point [17]. RDO that can improve the quality of a product, by minimizing the effect of variations without eliminating the causes, is another paradigm for designs under uncertainties. Du et al. [16] propose an integrated framework for design optimization under uncertainty that takes both the robustness of the objective and the probability of the constraints into account, so the RDO and RBDO can be achieved simultaneously.

Some studies of suspension systems based on RBDO or RDO have been presented in references. The work of [18] proposed a RBDO method considering the design variables as random variables, and the reliability of the suspension performance was quantified by the kinematic and compliance characteristics. The [19] studied a robust design methodology, in which a multi-objective evolutionary algorithm (MOEA) was applied to a passive suspension system of a linear quarter car. Robustness indexes have been analytically derived using the first-order Taylor approximation, thus allowing the robustness of each objective function to be integrated into the design process. Kim et al. [20] presented a robust design optimization for suspension systems, taking into account the kinematic behaviors influenced by bush compliance uncertainty. The variances of design goals are obtained through sampling the RBF metamodels, and a sequential approximation optimization technique is used to solve a robust design problem for the suspension system. Besides the RBDO and RDO, some other uncertain methods are also proposed for the analysis of vehicle dynamics, such as the polynomial chaos method [21], Monte Carlo method [22,23] and so on.

All the previous RBDO and RDO methods require the accurate probability information for the uncertain variables. However, it is generally expensive and time-consuming, and sometimes even impossible to get sufficient information to determine exact probability distribution functions, due to the complexity of engineering problems. Furthermore, Beb-Haim and Elishakoff [24] have shown that even small variations deviating from the real distributions may cause relatively large errors to the probability in the feasible region of the design space, and then may result in unreliable results of the optimization. As a result, the probabilistic methods for engineering problems with uncertainties may experience difficulty due to the absence of complete information. Recently, non-probabilistic methods [25,26] have provided alternative and useful supplements to the probability methods.

In particular, the interval method [22,23] has experiencing popularity, because it makes it possible to effectively express the uncertainties for uncertain-but-bounded parameters. Interval uncertain methods only require the lower and upper bounds of the uncertainty parameters, without necessarily knowing the precise distribution function. The determination of lower and upper bounds for an uncertain variable will be much easier than the identification of a precise probability distribution. For vehicle suspensions, the positions of hardpoints, spring stiffness and damping rate may vary around their nominal values due to production tolerances and wear, ageing, etc. [23]. The bounds of hardpoints positions can be extracted easier than the statistical information.