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Vibration of carbon nanotube reinforced composite beams based on the first and third order beam theories ☆

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ABSTRACT

This paper investigates the linear free vibration of nanocomposite beams reinforced by single-walled carbon nanotubes (SWCNTs). Two types of CNT reinforced beams, namely uniformly distributed CNT reinforced (UD-CNT) beams and functionally graded CNT reinforced (FG-CNT) beams, are considered. It is assumed that the SWCNTs are aligned along the beam axial direction and the distribution of the SWCNTs may vary through the thickness of the beam. The virtual strain and kinetic energies of the FG-CNT composite beam are obtained using the classic variational method of Hamilton's principle and then solved by the p-Ritz method. Vibration frequency parameters for the FG-CNT beams based on the first order and third order beam theories are presented and the effects of CNT filler volume fraction, distribution, beam span to depth ratio and end support conditions on the free vibration characteristics of the beams are discussed. Comparison studies for UD-CNT and FG-CNT beams based on the first order and the third order beam theories are also performed and the differences in vibration frequencies between these two theories are highlighted.

1. Introduction

Over the last several decades, fibre reinforced composites have found numerous applications in various industry sectors and have shown great superiority over conventional structural materials such as steel and alloys. Among many, glass fibre or carbon fibre reinforced polymers have been widely used on forming the fuselage of jumbo jets, blade of wind turbines, radar bonnets, cooling towers and racing cars, etc. The merits are obvious in terms of high strength–weight ratio, high stiffness–weight ratio and economic benefits. However, the inadequacies are also looming. Firstly, this type of fibre reinforced composites often is produced as laminated composites with different fibre orientations at different layers, and the mismatching of the material properties may cause delamination. Secondly, because of its relatively high volume fraction of fillers, to some extent, debonding or microscopic defects between fibres and matrix are prevalent. As a consequence, the overall performance of structural elements is degraded [1].

With the rapid progress of advanced nano-material technology, composites reinforced by nanoscale fillers have emerged as an alternative answer to the next generation of advanced materials. Unlike their micron scale counterparts, the physical properties of nanocomposites can be altered at an extremely low weight percentage of nanofillers. While for a given volume

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fraction, the interfacial regions between nanoparticles and matrix are significantly higher [2]. Moreover, the miniaturisation of devices can also be achieved by using nanocomposites, which may provide the next generation material solution used in the micro or nano electromechanical systems (MEMS or NEMS). Among those, carbon nanotube (CNT) reinforced nanocomposite is one of the most actively researched subjects due to its extraordinary mechanical, electrical and thermal properties. Aligned CNT array, thin films or wires, once incorporated into the matrix, not only enhances the mechanical properties along the alignment direction, but also affects the thermal and electrical conductivity of nanocomposites at certain direction. Recently, Shen [3] incorporated the concept of functionally graded materials (FGM) to the nanocomposites by varying the volume fraction of aligned CNTs along the thickness direction of plates. It can be foreseen by this mean the final structural element will have its unique properties that can be tailored in terms of vibration control, durability, electrical or thermal conductivities.

At nanoscale, due to the experimental difficulty, one of the most effective computational approaches for obtaining the mechanical properties of nanocomposites is the molecular dynamics (MD) simulation. The mechanical properties obtained by MD simulation can be utilised as a basis while the micromechanical approach can then be applied to analyse the microscopic material properties of nanocomposites. Bohlen and Bolton [4] studied the mechanical properties of SWCNT–Poly (vinylidene fluoride) composites using MD simulation and confirmed the alignment of the CNTs in the composite can have significant impact on the Young's modulus of the composite in that direction. Han and Elliott [5] modelled the SWCNT–Poly (methyl methacrylates) and SWCNT–PmPV system and concluded that the general rule of mixtures is not accurate for composites with strong interfacial interactions. Griebel and Hamaekers [6] simulated the SWCNT–Poly (ethylene) composite and suggested that an extended rule-of-mixtures should be used in the case of short nanotube reinforced composites. In addition to that, Wang and Zhang [7] assessed the elastic properties and the effective wall thickness of the SWCNT and concluded that the wall thickness of the SWCNT should be less than 0.142 nm. Therefore the modulus of the nanotube should be even higher than previously reported with the wall thickness of 0.34 nm. The mechanical property simulation of the SWCNT using finite element method can also be found in the research work by Lu et al. [8] and Giannopoulos et al. [9].

Shen and his colleagues conducted a series of research on bending, vibration and buckling problems of the functionally graded CNT reinforced plates and shells using the two step perturbation technique [3,10–17]. The governing equations were based on a third order shear deformation theory with von Kármán type of kinematic nonlinearity. The research showed that a functionally graded distribution could have significant effect on the load-bending moment curves, vibration characteristics as well as the buckling and thermal buckling behaviours of the plates and shells. Zhu et al. [18] studied the static and free vibration of FG-CNT plates using the finite element method under the framework of first order shear deformation theory. They concluded that for the free vibration analysis, both the CNT volume fraction and the width-to-thickness ratio have pronounced effect on the natural frequencies and vibration mode shapes of the CNT reinforced composite plates. Rafiee et al. [19] analysed large amplitude free vibration of FG-CNT beams with surface bonded piezoelectric actuator layers based on Euler–Bernoulli beam theory. Ke et al. [20] investigated the geometric nonlinear free vibration of FG-CNT beams based on the first order shear deformation theory. Later, Ke et al. [21] considered the size effect and investigated the bending, buckling and vibration behaviour of functionally graded micro plates.

Most of the vibration systems in the engineering applications are highly nonlinear and there exist many nonlinear factors such as the geometric [15,20], material nonlinearities or nonlinear forcing and damping mechanisms. However comparing with the much more complex nonlinear theory, a linear theory remains the cost-effective way which gives the first-cut analysis and captures important information in the vibrating system. The linear free vibration theory assumes that the vibration amplitude of the beams is small and material constitutive relation is in the linear elastic range, and these assumptions are valid for many engineering applications. The linear free vibration behaviours of FG-CNT beams are the focus of the current study.

The model based on first order shear deformation theory requires shear correction factors which may take the values of 5/6 and $\pi^2/12$ or be a function of the Poisson ratio [22]. However, in practice and in some of the reviewed literatures it has usually been assumed to be 5/6. This value may not be appropriate for the FGM as the material properties vary through the thickness of structures [23]. The third order shear deformation theory [24] assumes parabolic transverse normal deformation. As a result, no shear correction factor is needed. Besides, better solution can be attained kinematically. In the current study, the energy functional of functionally graded CNT nanocomposite beams for linear free vibration is obtained using classic variational method and based on both the first order and third order shear deformation theories. The eigenvalue equation is solved by the p-Ritz method [22]. Comparison studies between the first order and third order beam theories will be presented and discussed. Based on the third order shear deformation theory, vibration parameters of uniformly distributed CNT (UD-CNT) beams and functionally graded CNT (FG-CNT) beams under various combinations of geometry, boundary, CNT volume fraction and distribution are presented and discussed in details.

2. Mathematical formulation

2.1. Problem definition

As shown in Fig. 1(a), consider a composite beam system of length L, width b, thickness h along x, y and z directions, respectively. Three types of aligned CNT reinforced beams are considered, namely the uniformly distributed CNT beams