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A response-surface-based structural reliability analysis method by using non-probability convex model ☆

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ABSTRACT

Due to its weak dependence on the amount of the uncertainty information, the non-probability convex model approach can be used to deal with the problems without sufficient information. In this paper, by integrating the response surface (RS) technique with the convex model approach, a new structural reliability analysis method is developed for many complex engineering problems with black-box limit-state functions. Using the newly developed correlation analysis technique for non-probability convex model, the multi-dimensional ellipsoid is efficiently constructed to characterize the uncertain parameters. A quadratic polynomial without cross terms is adopted to parameterize the black-box limit-state function, based on which the functional values as well as the first-order gradients can be explicitly calculated. At each iteration, the created RS is combined with the *i*HL-RF algorithm to obtain an approximate reliability index. A sequential procedure is subsequently formulated to update the RS and hence improve the precision of the reliability analysis. Four numerical examples and one engineering application are investigated to demonstrate the effectiveness of the presented method.

1. Introduction

Uncertainties associated with geometric tolerances, material properties, boundary conditions and etc. widely exist in practical engineering problems. The probability approach is the most popular way to quantify uncertain parameters and whereby conduct the reliability analysis [1–3]. In the past several decades, various probabilistic reliability analysis methods were well developed, some of which have been successfully applied to many engineering problems. The most commonly used reliability approaches consist of Monte Carlo simulation (MCS) [4,5], Latin hypercube sampling (LHS) [6,7], the first-order reliability method (FORM) and the second-order reliability method (SORM) [8–10], etc.

An alternative approach for uncertainty quantification is the non-probabilistic convex model, in which the uncertain parameters are modeled as a multi-dimensional ellipsoid or a solid box. This quantification method was originally proposed by Ben-Haim and Elishakoff [11] and Ben-Haim [12], and then explored by many other researchers. Compared to the probabilistic approach, weak dependence on the sample information makes the convex modeling approach more and more attractive. An "uncertain triangle" was used to describe the relations between the existing three uncertainty analysis

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approaches including probability, fuzzy set and convex model [13]. A comparison between convex model and interval analysis method was performed to predict the effects of uncertain-but-bounded parameters on the buckling of composite structures [14]. An interval analysis technique was proposed to calculate the static and dynamic responses of uncertain structures based on an improved first-order Taylor interval expansion [15]. An interval sensitivity analysis approach was suggested to measure the individual influence of uncertain inputs on the range of the structural response [16]. A modified order relation of convex model was developed to estimate the reliability level and applied to structural reliability design [17,18]. Special formulations were developed for confidence robust structural design optimization under non-probabilistic stiffness and load uncertainties, respectively [19]. A non-probabilistic reliability index was defined as the minimal distance in the standard convex space [20,21], and a corresponding efficient solution algorithm is subsequently formulated by extending the FORM technique in traditional probability analysis into the presented convex model analysis [22]. In the above-mentioned analysis, the non-probabilistic reliability index is regarded as a possible maximum uncertain extent that the structure can bear rather than the probability meaning. The non-probabilistic reliability technique has been successfully applied to some relatively simple problems with analytical expressions, however, there still exist several challenging difficulties to solve practical engineering problems. One important issue among the difficulties is that most practical engineering problems are based on complex simulation models such as finite element method (FEM), computational fluid dynamics (CFD), etc. And these simulation models could be formulated only using implicit functions with respect to the concerned parameters. Thus, to carry out an efficient reliability analysis, corresponding parameterizations seem necessary for these implicit models. For this kind of problems, response surface (RS) is perhaps a better choice because RS can be capable of approximating the black-box limit-state functions and furthermore filtering the numerical noises [23]. Additionally, the RS seems helpful for reducing the computational cost brought by the time-consuming simulations.

In this paper, a RS-based non-probabilistic reliability analysis technique is proposed for black-box limit-state function. The structure of this paper is organized as follows. Section 2 addresses the non-probabilistic reliability index by using convex model. The proposed RS-based reliability analysis technique is presented in Section 3. Four numerical examples and a practical engineering problem are investigated in Section 4. Section 5 draws some conclusions on the present method.

2. Non-probabilistic reliability index by using convex model

Consider the following limit-state function with n uncertain parameters

$$g(\mathbf{X}) = 0, \quad X_i = 1, 2, \ldots, n, \tag{1}$$

where the system state is divided into two regions, one is the safety region for $g(\mathbf{X}) \geqslant 0$ and the other is the failure region for $g(\mathbf{X}) < 0$. In this paper, the multi-dimensional ellipsoid is adopted to quantify the parameter uncertainty, which is one of the most widely used convex models. All of the possible combinations of the uncertain parameters will then fall into a following multi-dimensional ellipsoid [24]

$$(\mathbf{X} - \mathbf{X}^0)^{\mathrm{T}} \mathbf{G} (\mathbf{X} - \mathbf{X}^0) \leqslant 1, \tag{2}$$

where the vector $\mathbf{X}$ collects all the uncertain parameters, and $\mathbf{X}^0$ is the midpoint of $\mathbf{X}$. $\mathbf{G}$ is the characteristic matrix of the ellipsoid, which is also a symmetric positive-definite real matrix determining the size and orientation of the ellipsoid.

For computing the reliability index [20,21], the uncertain parameters $\mathbf{X}$ (x space) should be firstly transformed into a set of dimensionless variables $\mathbf{U}$ (u space)

$$U_i = \frac{X_i - X_i^c}{X_i^w}, \quad i = 1, 2, \ldots, n, \tag{3}$$

where X_i^c and X_i^w are the corresponding midpoint and radius of each uncertain-but-bounded parameter

$$X_i^c = \frac{X_i^L + X_i^R}{2}, \quad X_i^w = \frac{X_i^R - X_i^L}{2}, \quad i = 1, 2, \ldots, n, \tag{4}$$

where X_i^R and X_i^L are upper and lower bounds of each uncertain parameter X_i, respectively.

Based on this transformation, an equivalent ellipsoid can be obtained in u space

$$\mathbf{U}^{\mathrm{T}} \mathbf{W} \mathbf{U} \leqslant 1, \tag{5}$$

where $\mathbf{W}$ is the characteristic matrix of the ellipsoid in u space.

Cholesky Decomposition is then performed to the $\mathbf{W}$ matrix

$$\mathbf{W} = \mathbf{L}_0 \mathbf{L}_0^{\mathrm{T}}, \tag{6}$$

where $\mathbf{L}_0$ is a lower triangular matrix, and the superscript T represents a matrix transpose.

Another linear transformation is also introduced

$$\boldsymbol{\delta} = \mathbf{L}_0^{\mathrm{T}} \mathbf{U}. \tag{7}$$