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# The time constrained maximal covering salesman problem

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ABSTRACT

We introduce the time constrained maximal covering salesman problem (TCMCSP) which is the generalization of the *covering salesman* and *orienting* problems. In this problem, we are given a set of vertices including a central depot, customer and facility vertices where each facility can supply the demand of some customers within its pre-determined coverage distance. Starting from the depot, the goal is to maximize the total number of covered customers by constructing a length constrained Hamiltonian cycle over a subset of facilities. We propose several mathematical programming models for the studied problem followed by a heuristic algorithm. The developed algorithm takes advantage of different procedures including *swap*, *deletion*, *extraction-insertion* and *perturbation*. Finally, an *integer linear programming* based improvement technique is designed to try to improve the quality of the solutions. Extensive computational experiments on a set of randomly generated instances indicate the effectiveness of the algorithm.



## 1. Introduction and literature review

The concept of *covering* is used in many practical applications, introduced in the literature. Taking into account the limited resources, in many real applications covering the demand of all customers is not possible. As a result, in this situation the goal is to propose the best strategy to maximize the amount of covered demand by taking into account the limitations of the available resources. Church and Revelle [1] introduced the maximal covering location problem (MCLP) in which we are given a set of potential facility vertices and a set of customers. In particular, each facility $i$ can cover a set of customers within a given service distance (time) $r_i$ and the goal is to open a fixed number of facilities, say $p$, to cover the maximum amount of demand [1]. Later in 1984, Church introduced the planar case of the MCLP in which the facility and customer vertices are not discrete locations and could be assumed as continues regions on the plane [2]. Exact and heuristic algorithms are proposed by researchers to solve this problem [3,4].

Introduced by Current and Storbeck [5], in the capacitated maximal covering location problem (CMCLP) we are given a capacity restriction on the size of each facility while the other assumptions are the same as those proposed for the MCLP.

Knowing the minimum number of facilities, say $p$, that are sufficient to cover the demand of the customers in a given region, there may be multiple solutions with the same objective value $p$. In the maximal covering with mandatory closeness constraints [6], it is not necessary for the demand of each customer to be satisfied using its nearest established facility, and the objective is to cover the demand within a desired distance of the selected facilities.

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In the covering salesman problem (CSP) [7,8], we are given a set of customers where each customer $i$ can cover a subset of other customers located within its pre-determined distance $r_i$. The objective of the CSP is to construct a minimum length Hamiltonian cycle over a subset of customers, in which those not visited by the tour, have to be within a given coverage distance of at least one customer visited on the tour.

Gendreau et al. [9], proposed a generalization of the CSP, called the covering tour problem (CTP). In this problem, the set of vertices is divided into three different groups, namely $Y_1, Y_2$ and $Y_3$. The vertices belonging to $Y_1$ are those that must be visited by the tour, while $Y_2$ consists of the vertices which can be visited, and $Y_3$ contains those that must be covered (i.e. to be within a specified distance of a visited vertex). The goal of the CTP is to construct a minimum length Hamiltinian cycle over all vertices in $Y_1$ and a subset of vertices in $Y_2$ in which all vertices in $Y_3$ are covered by the visited ones.

The maximum covering tour problem (MCTP) was introduced to minimize the total uncovered demand of a given set of customers through constructing a minimum length tour over a set of $p$ customers of the solution. Here, the assumption is that each customer is able to cover a set of other customers which are within its pre-determined distance (time) [10].

Finally, Golden et al. [11] developed a generalization of the CSP in which the assumption of visiting or covering each vertex for at least once, is not valid. In particular, in this problem each vertex $i$ has to be covered or visited at least $k_i$ times.

Recently, a very nice survey paper on the covering problems in facility location has been published by Farahani et al. [12]. To study the characteristics of the other problems related to this area, we just refer the readers to this paper [12].

There is an abundant body of literature, dedicated to the prize collecting traveling salesman problem (PCTSP) [13]. The PCTSP is to construct a minimum cost Hamiltonian cycle over a subset of vertices in a given network with a side constraint. Essentially, visiting each vertex leads to a profit and failing to visit a vertex will cause a pre-specified penalty. The goal of the PCTSP is to minimize the total tour length while collecting a prescribed amount of prize [13].

Golden et al. introduced the orienteering problem (OP) in which we are given a set of vertices, each having a given score. The goal of this problem is to design a length constrained path over a subset of the vertices where the total collected score is maximized [14]. Among different applications proposed in the literature for the OP, we can refer to the fuel delivery [14], traveling salesman problem with not enough time to visit all the vertices [15], ring design in building telecommunication networks [16] and mobile tourist guide [17]. For a recent survey on the OP we refer the interested readers to the paper by Vansteenwegena et al. [18].

## 2. Problem description

We propose a natural generalization of the CSP and the OP, called the time constrained maximal covering salesman problem (TCMCSP). In this problem, we are given a central depot, a set of facility and a set of customer vertices where each facility can cover a subset of customers which are located within its pre-determined coverage distance (time). Starting from and ending at the depot, the goal is to construct a length (time) constrained Hamiltonian cycle over a subset of facility vertices, while maximizing the total number of covered customers.

The introduced problem has potential real-world applications. Essentially, some applications proposed for the OP, could be easily extended to the case of the TCMCSP. As an example, when designing mobile telecommunication networks, suppose each tower can cover a given number of customer locations, while different towers can cover the same demand zones, simultaneously. A natural generalization of the OP introduced in [16] could be easily extended to the case of the TCMCSP in which the goal is to construct a length constrained cycle over a subset of towers in which the total covered demand is maximized. In the following we introduce some applications of the TCMCSP in different areas of combinatorial optimization problems.

The introduced problem may occur in the rural health care delivery, when we have an emergency situation for preventing an epidemic disease or assuring public health and security. Assume that a health care team should travel within the rural areas to do vaccination immediately, while in each area the people are requested to travel to their nearest tour stop. In this situation, we should decide on the selected locations and the order of visiting them by the tour with the goal of maximizing the covered population within a given time limit.

We can also refer to the disaster situations caused by natural or man-made events, like an earthquake or war, when we should provide the population of an affected region with the necessary needs such as food, water and blanket. Since there is a limited time in which the injured people can live without the necessary drugs or initial humanitarian aids, we should provide them with the materials very urgently. So, in all of these applications, we have to maximize the number of people who receive the services within a given time limit.

Also, suppose a politician has decided to run for the elections. He may have a limited time to visit different cities to discuss his programs. Moreover, different regions may have much different profits for him, meaning that by visiting each place just a given number of people can benefit from his programs. So, the question would be which cities must be visited to cover the maximum number of people, while be respectful to the maximum time limit.

Having a time limit of 25 units, Fig. 1 illustrates an example of the introduced problem in which starting from and ending at the depot, four facility vertices are visited by the tour and totally eleven customers are covered. In this figure, the number next to each edge is the length corresponding to that edge.

The main contributions of this paper are twofold: (1) we provide the detailed description of the TCMCSP and provide some mathematical models; (2) we develop an ad hoc heuristic, designed for the studied problem that is able to find good quality solutions in a short computing time which is a desirable solution method, based on the nature of the problem.