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# The backward group preserving scheme for multi-dimensional nonhomogeneous and nonlinear backward wave problems

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ABSTRACT

In this study, we utilize a backward group preserving scheme (BGPS) to cope with the nonhomogeneous as well as nonlinear backward wave problems (BWPs). Because the solution does not continuously count on the given information, the BWP is well-known to be seriously ill-posed. When six numerical instances are examined, we reveal that the BGPS is capable of tackling the nonhomogeneous and nonlinear BWPs. Besides, the BGPS is also robust enough against the perturbation even with the boisterous final data, of which the numerical results are rather accurate, effective and stable.



## 1. Introduction

Many researchers are concerned with the forward wave problems for a long term numerical computation; however, only few researchers are interested in the backward wave problems (BWPs) because of their inherent ill-posed properties. There are many schemes for coping with the forward wave problems, for example, the finite difference schemes [1], the invariant imbedding method [2], the modal analysis [3], a mixed finite element approximation and domain decomposition [4], the variational iteration method [5,6], the method of lines [7], the homotopy perturbation method and a modified homotopy perturbation method [8], a complete symmetry analysis [9], the method of bifurcation theory of dynamic systems [10], a second-order approximate symmetry classification [11], the bifurcation theory of planar dynamical systems [12], the existence of a time-periodic solution of a two or three-dimensional nonlinear wave equation [13], the local discontinuous Galerkin methods for two classes of nonlinear wave equations [14], the method of separation of variables for the nonlinear wave equation [15], the Fourier pseudospectral method, the finite difference method and the fast Fourier transform for the nonlinear wave equation [16], the method based on KAM theory for the nonlinear wave equations under quasi-periodic forcing [17], the analysis of nonlinear wave equations by using the sharp interface limit [18], and the localized differential quadrature method and the fourth-order Runge–Kutta method for wave propagation in a poroelastic medium [19].

For the homogeneous backward wave problems as described by Ames and Straugham [20], it has significant applications in geophysics and the optimal control theory. After that, Lesnic [21] has addressed the Adomain decomposition method to tackle the BWP, and indicated that for the forward problem the convergence of the Adomain decomposition method is faster than that for the backward problem. Then, Liu [22] utilized the eigenfunctions expansion techniques and the separating

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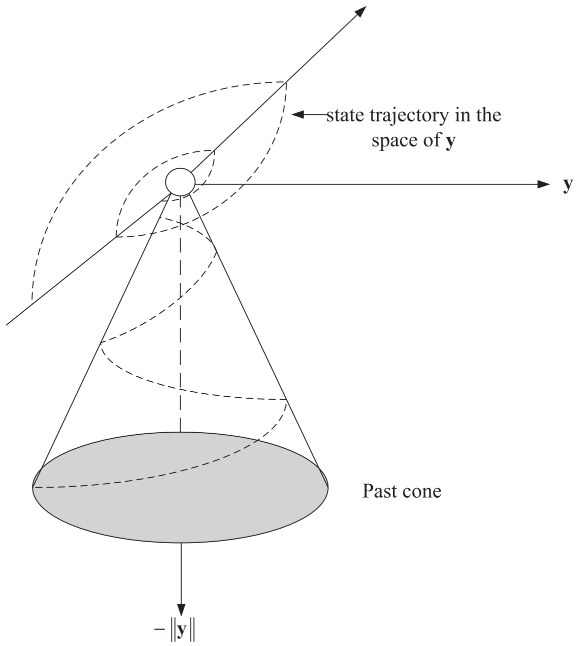


**Fig. 1.** The construction of deleted cones in the Minkowski space for backward problems signifies a conceptual breakthrough. The trajectory remarked in the state space **y** is a parallel projection of the trajectory in the null cones along the ‖**y**‖-axis.

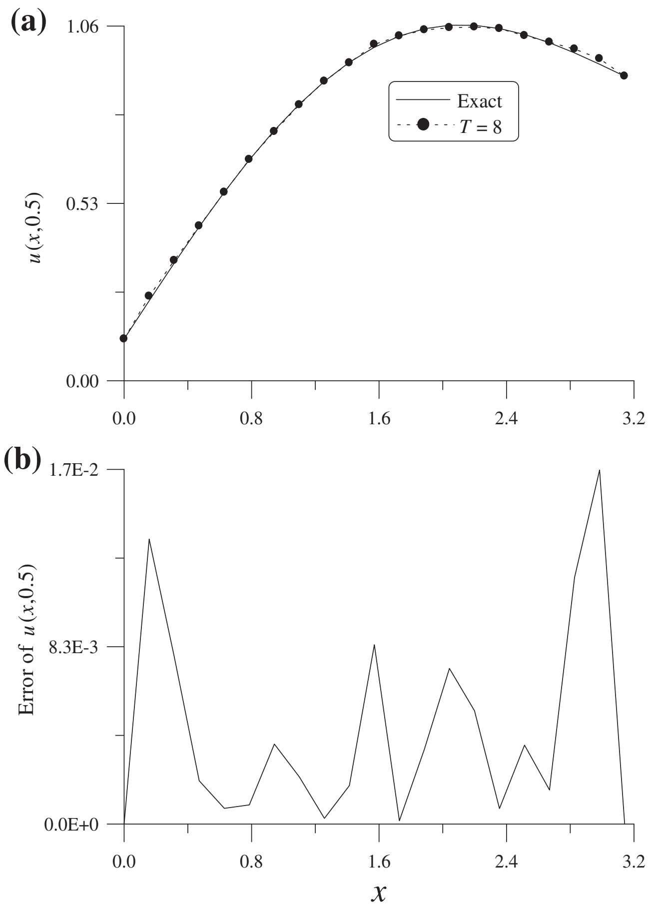


**Fig. 2.** Comparisons of the exact solutions and numerical solutions with final time $T = 8$, and the corresponding numerical errors.