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Analytical solution of coupled soil erosion and consolidation equations by asymptotic expansion approach

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ABSTRACT

In this work, one-dimensional approximation of internal erosion taking place in a soil made from sand and clay mixture was considered. The clay phase that is susceptible to experience erosion under water flow discharge was assumed to be small. A new erosion law fixing the initiation threshold of erosion and integrating the effect of soil consolidation on internal erosion was proposed. Conversely, the effect of erosion on elastic soil deformation was also integrated through damage mechanics concepts. Asymptotic expansion of the coupled equations in terms of a perturbation parameter linked to the total amount of internal erosion that is likely to occur has been performed. This has enabled to view the internal erosion phenomenon occurring inside the soil as a perturbation affecting the classical soil consolidation equation, and further to evaluate the critical discharge gradient for which internal erosion starts. Equations at order zero that are provided by the asymptotic expansion were exactly integrated while an adequate finite difference scheme was introduced to solve the equations at order one. A parametric study was conducted after that in order to assess effects of the main factors on internal erosion and soil deformation.

1. Introduction

Stability of soils under buildings or hydraulic infrastructures such as levees or dams is of great importance in practice. Even if initially design of foundations meets well safety criteria, problems of durability can arise. One of the most important sources of degradations yielding to instabilities of foundations is internal erosion. This phenomenon is related to the migration of soil particles through the soil matrix by suffusion or piping. This phenomenon is generated by water seepage as the hydraulic gradient reaches a critical value. It starts as suffusion of soil fines that takes place inside the foundations of hydraulic retaining structures such embankment dams and levees, then progresses until reaching the advanced stage of piping. This last step consists of regressive erosion which develops from downstream of the critical seepage line towards its upstream with the formation of a continuous pipe, [1].

The risk threatening hydraulic infrastructures due to internal erosion is aggravated by the sudden character of this phenomenon. No external sign enabling its early detection is visible before piping occurrence. A dam may breach in fact only a

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few hours after first evidence of piping has been stated. Understanding the early stages of internal erosion during the suffusion phase is of huge interest from the practical point of view. Predicting internal erosion can give in this way the possibility to perform in better way health monitoring of hydraulic infrastructures through assessing active protection of these vital facilities.

As pointed out in [2], suffusion occurs when a soil satisfies the following criteria: it has a structure that possesses both a coarser fraction containing voids and a finer fraction; the loose particles constituting the finer fraction are smaller than the constrictions formed by the coarser fraction, and finally water flow velocity is strong enough to transport the fines through the voids.

Several physical models that describe internal erosion have been derived in the last decades [3–6]. In most of the proposed models, a continuum approach is adopted. The soil is viewed as a three-phase medium with the representative volume element consisting of solid particles, discharging fluid and fluidized grains which are extracted by erosion from the soil skeleton [3]. Internal erosion is activated locally by a seepage flowing with a supercritical gradient and was recognized to be strongly coupled with the hydro-poro-mechanical behaviour of the soil. The effect of internal erosion on the soil was integrated by introducing a tangent elastic plastic porosity matrix that affects soil deformation and by taking into account variations of permeability as function of porosity [7]. Based on experimental evidence that states that mass production of eroded particles tends to decrease over time, an erosion law had been suggested by assuming that the production rate of fluidized grains is proportional to the hydraulic gradient discharge [6–8,3]. Under this form, the equation of erosion was coupled to Darcy equation and thus to consolidation phenomenon taking place in the soil.

Most of the basic ideas existing in the previous literature dealing with modelling internal erosion are carried out in the actual study. But, since focus will be done on the early stage of erosion, the soil skeleton is assumed to deform elastically in the vicinity of an equilibrium point that has been reached during the pre erosion stage. In addition to that, variations of porosity resulting from erosion are supposed to yield damage of the soil. Even if the determination of the thermodynamical potential associated to this damage mechanism is still an open problem, the soil Young's modulus is allowed to vary according to the classical law of damage mechanics [10,11]. Using an extension of the erosion law that was first formulated in [7], the authors of the present work have given one-dimensional approximation of internal erosion equations [11]. This was performed in the special case of soil samples made from controlled mixtures of sand and clay but containing a small proportion of clay that is susceptible to experience erosion. The equations governing internal erosion problem were found to have the general form of coupled equations that include erosion, poro-elastic consolidation and transport of eroded particles.

The big problem with the obtained equations is that they are instable and their direct numerical integration through the standard finite element method has given rise to poor results. This has been well assessed earlier, since numerical models based on coupled formulations even in the simplest soil consolidation problem without any erosion were recognized to become ill conditioned when the soil permeability is small or soil deformability is high. To overcome these numerical difficulties special treatment was proposed in the literature [12] by defining a new element through an averaging technique.

This is why even for the decoupled problem analytical approaches are still extensively used to deal with soil consolidation. Xie et al. [13] have developed a general approximate analytical solution for one-dimensional consolidation with consideration of the threshold gradient under a time-dependent loading, and the approach was found to be quite reasonable. Considering large strain consolidation, Xie et al. [14] have given an explicit analytical solution for the one-dimensional problem. Comparisons that were made with the classical small strain theory have shown that the average degree of consolidation is modified. Using Laplace transform, Qin et al. [15] have presented an analytical solution to one-dimensional consolidation in unsaturated soils with the applied vertical loading varying exponentially with time.

Due to the complexity of the non-linear consolidation of soft clay, other robust approaches were proposed like the differential quadrature method which was used in [16] to solve one-dimensional nonlinear consolidation problem for a multi-layered soil. Zheng et al. [17] have used also this method to implement solution of one dimensional nonlinear consolidation equation with various boundary conditions, and the obtained numerical results compared satisfactorily with analytical solutions.

In the framework of fully coupled equations that comprise both internal erosion and consolidation of soils, literature dedicated to this subject is actually rare. Among the few works in this field, Wang and Wan [18] have observed numerical difficulties while using the standard finite element method to integrate the problem. These instabilities had taken the form of wiggles that had arisen in the case of high gradient conditions that follow large variation of the local field variables. In order to capture the associated local sharp changes, the field variables that enter the differential equations modelling the problem were averaged around material points under consideration.

In a previous work [11], the authors have introduced an asymptotic expansion approach in order to solve in a stable way the one dimensional soil coupled internal erosion-consolidation equations. Within the context of this approach, the internal erosion consolidation problem is viewed to be as the classical consolidation problem perturbed by porosity variations. This suggested performing asymptotic expansion of the obtained equations in terms of a scale parameter chosen to be the magnitude of expected porosity variations. The classical consolidation problem appeared like this to coincide with the degenerated erosion free problem. Using the asymptotic expansion, the problem was shown to have a closed form analytical solution.

To reproduce experimental evidence stating that erosion appears only when the seepage velocity is supercritical, a further extension is considered here for the erosion law which was first introduced in [11]. A threshold criterion for minimum gradient required to initiate erosion is formulated. The asymptotic expansion technique is used with some modifications that