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Adaptive synchronization of dynamical networks via states of several nodes as target orbit

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ABSTRACT

In this paper, based on the invariance principle of differential equation, a simple adaptive control method is proposed to synchronize the dynamical networks with the general coupling functions. Comparing with other adaptive control methods, the weighted average of a few nodes' states is used as target orbit to design controller. To show the effectiveness of proposed method, some numerical simulations are performed.

1. Introduction

Since the pioneering work of Pecora and Carroll on synchronization [1,2], synchronization has been an important topic in nonlinear science. Up to now, many approaches have been proposed to synchronize two coupled chaotic systems, such as nonlinear control [3], adaptive control [4–6], active control [7,8], sliding mode control [9,10] and so on.

In fact, many natural and man-made systems such as biological systems, neural systems and social systems, may be described by dynamical networks which consist of many coupled nonlinear systems with different complex coupling structures [11,12]. Because many observed natural phenomena can be well explained by the synchronization of dynamical networks, many attentions have been attracted from scientists in different areas. Up to now, some effective methods have been proposed to analyze the synchronization of dynamical networks, such as master stability function method [13], connection graph stability method [14] and the new method proposed by Lu [15]. With these methods, the conditions of synchronization of dynamical networks with different structures have been studied [13,15,14,16–18]. On the other hand, the real-word networks are usually disturbed by different factors, so it is difficult for real-word networks to achieve synchronization spontaneously. To achieve synchronization of real-word networks, some control methods have been proposed. In Ref. [19–22], authors discussed how to synchronize the networks by pinning control. In Ref. [19,21], authors discussed two different pinning strategies: randomly pinning and selective pinning based on the degree of nodes, and found that pinning strategies by highest degree is better than randomly pinning. In Ref. [20], authors found that a single controller can synchronize the network via a solution of the uncoupled system as target orbit, if the coupling strength is chosen suitably. Due to the simple structure of impulsive control, the synchronization of dynamical networks by impulsive control

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was discussed in Ref. [23,24]. As the ability of quick automatic response, some effective adaptive control methods were proposed in Ref. [25–30]. These control methods are very effective, which can synchronize the complex networks with chaotic nodes without knowing the concrete network structures. However, most of above controllers are designed by a solution of uncoupled system. So we must know a solution of the node system in advance, if we use those control schemes. Usually, we can not obtain a solution of the node system, for example, the network does not have an isolate node, and the concrete structure of the node system is unknown. To solve the above problems, we have proposed an adaptive control method using average of all nodes' states as target orbit [31–33]. Recently, authors synchronize the networks by impulsive control using weighted average of a few nodes' states as target orbit [34]. Inspired by their work, we will prove that synchronization can also be achieved by adaptive control using above target orbit, so the method in this paper is an improvement of method in Ref [31–33]. The advantages of this improvement will be discussed in Remark 1.

2. Adaptive control scheme

Considering the dynamical networks consisting of m coupled nodes with general coupling functions

$$\dot{\boldsymbol{x}}^i = \boldsymbol{f}(\boldsymbol{x}^i) + \boldsymbol{g}^i(\boldsymbol{x}^1, \boldsymbol{x}^2, \ldots, \boldsymbol{x}^m), \quad i = 1, 2, \ldots, m, \tag{1}$$

where $x^i = (x_1^i, x_2^i, \ldots, x_n^i)^T \in R^n$ is state vector of i-th node, $\boldsymbol{f}(\boldsymbol{x}) = (f_1, f_2, \ldots, f_n)^T : R^n \to R^n$ is a nonlinear vector function describing the dynamics of an isolated node, $\boldsymbol{g}^i(\boldsymbol{x}^1, \boldsymbol{x}^2, \ldots, \boldsymbol{x}^m) : R^{m\times n} \to R^n$ $(i = 1, 2, \ldots, m)$ are unknown nonlinear coupling functions.

The aim of this paper is to design an effective control method to synchronize the dynamical networks, so we add a controller $\boldsymbol{u}^i = (u_1, u_2, \ldots, u_n)^T$ on each node.

$$\dot{\boldsymbol{x}}^i = \boldsymbol{f}(\boldsymbol{x}^i) + g^i(\boldsymbol{x}^1, \boldsymbol{x}^2, \ldots, \boldsymbol{x}^m) + \boldsymbol{u}^i, \quad i = 1, 2, \ldots, m. \tag{2}$$

In Ref. [15], authors proposed a new method to studied stability of the synchronization manifold of dynamical network by $\boldsymbol{X}(t) = \sum_{i=1}^m \xi^i \boldsymbol{x}^i$ ($\xi^i \geqslant 0$ and $\sum_{i=1}^m \xi^i = 1$) which can be regarded as a projection of state of network on the synchronization manifold. Inspired by above study, the authors in Ref. [31–33] have proposed an adaptive method to synchronize the dynamical network via average of all nodes' state as target orbit. In this paper, the target orbit is replaced by $\boldsymbol{X}(t) = \sum_{i=1}^m \xi^i \boldsymbol{x}^i$ (Here $\xi^i \geqslant 0$, and $\sum_{i=1}^m \xi^i = 1$). The advantage of this improvement will be discussed in Remark 1. Obviously, $X(t)$ satisfies the following equation

$$\dot{\boldsymbol{X}}(t) = \sum_{i=1}^m \left[\xi^i \boldsymbol{f}(\boldsymbol{x}^i) + \xi^i \boldsymbol{g}^i(\boldsymbol{x}^1, \boldsymbol{x}^2, \ldots, \boldsymbol{x}^m) + \xi^i \boldsymbol{u}^i\right], \tag{3}$$

Defining the vectors

$$\delta\boldsymbol{x}^i = \boldsymbol{x}^i - \boldsymbol{X}(t), \quad i = 1, 2, \ldots, m, \tag{4}$$

where $\delta\boldsymbol{x}^i = (\delta x_1^i, \delta x_2^i, \ldots, \delta x_n^i)^T \in R^n$ is usually called synchronization error which describes deviation of the state of i-th node from $\boldsymbol{X}(t)$. The error vectors $\delta\boldsymbol{x}^i (i = 1, 2, \ldots, m)$ should satisfy the following equation

$$\delta\dot{\boldsymbol{x}}^i = \boldsymbol{f}(\boldsymbol{x}^i) + \boldsymbol{g}^i(\boldsymbol{x}^1, \boldsymbol{x}^2, \ldots, \boldsymbol{x}^m) + \boldsymbol{u}^i - \boldsymbol{G}(\boldsymbol{x}), \quad i = 1, 2, \ldots, m, \tag{5}$$

where $\boldsymbol{G}(\boldsymbol{x})$ is a vector function

$$\boldsymbol{G}(\boldsymbol{x}) = \sum_{i=1}^m [\xi^i \boldsymbol{f}(\boldsymbol{x}^i) + \xi^i \boldsymbol{g}^i(\boldsymbol{x}^1, \boldsymbol{x}^2, \ldots, \boldsymbol{x}^m) + \xi^i \boldsymbol{u}^i]. \tag{6}$$

Here, we should notice that $\delta\boldsymbol{x}^i$ $(i = 1, 2, \ldots, m)$ satisfy the following condition

$$\sum_{i=1}^m \xi^i \delta\boldsymbol{x}^i = 0. \tag{7}$$

Obviously, the stability of synchronization of network (2) corresponds to the stability of zero solution to system (5). So our aim is to design controllers $\boldsymbol{u}^i (i = 1, 2, \ldots, m)$ to make the system (5) be asymptotically stable at origin. Inspired by Ref. [4,31–33], we introduce the following adaptive-feedback controller $\boldsymbol{u}^i$

$$u_k^i = \varepsilon_k \left(x_k^i - \sum_{i=1}^m \xi^i x_k^i \right), \quad i = 1, 2, \ldots, m, \; k = 1, 2, \ldots, n, \tag{8}$$

where $\boldsymbol{\varepsilon} = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)^T \in R^n$ is adaptive feedback strength. The feedback strength $\boldsymbol{\varepsilon}$ will be adaptive update according to the following algorithm

$$\dot{\varepsilon}_k = -\gamma_k \frac{1}{m} \sum_{i=1}^m \left(x_k^i - \sum_{i=1}^m \xi^i x_k^i \right)^2, \quad k = 1, 2, \ldots, n, \tag{9}$$

where $\gamma_k (k = 1, 2, \ldots, n)$ are arbitrary positive constants.