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Short communication

A short note on the solution of a multi-effect evaporator system employed in pulp and paper industry

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ABSTRACT

In this short communication the solution of a steady state model of a sextuple evaporator system used in a paper industry has been presented. With the help of the residual values of model equations (a set of 12 nonlinear algebraic equations) it has been shown that the presently obtained results are superior to the previously published results. The importance of the number of significant digits in calculation are also briefly discussed.

1. Introduction

Multi-effect evaporator (MEE) systems are widely used in various chemical and allied industries for concentrating the dilute solutions and form the main part of the unit operations, e.g. sugar cane juice is concentrated in sugar industry, caustic soda solution is concentrated in caustic soda manufacturing plant and black liquor is concentrated in a pulp and paper industry. Due to their energy intensive characteristics these systems have been studied extensively and a lot of literature especially concerning the development of model and its simulation is available [1–5]. The modelling and simulation studies of these systems are carried out either to design a new system for a given output or to judge the performance of an existing one. In either case, the steady state models of these systems are developed by applying the mass and energy balances along with the relevant constitutive equations [1,2,5], and are represented by a set of nonlinear algebraic equations (AEs). As reported in literature, these equations pose several difficulties in obtaining their solutions since the resultant Jacobian matrix is ill-conditioned, and various specific methods are employed for overcoming them [2,3].

This short communication presents the steady state solutions of one such MEE system (a sextuple evaporator system) typically used in a paper industry and compares them with the recently published results of Kumar et al. [1]. It is shown that the presently obtained results are better as compared to those obtained by Kumar et al. [1] in the sense that the residual values of equations are very small in present case. The steady state model of the selected MEE system is represented by the following 12 nonlinear AEs (Eqs. (1)–(12)) in 12 unknowns ($A, W_{V0}, T_{V1}, T_{V2}, T_{V3}, T_{V4}, T_{V5}, X_2, X_3, X_4, X_5$ and X_6). In their study Kumar et al. [1] have solved this system of AEs by employing the inbuilt MATLAB command "fsolve' used for solving nonlinear AEs. For comparison purposes the results obtained by Kumar et al. [1] are reproduced in Table 1. In the present study, we too have employed the same MATLAB command for solving these 12 AEs; however, in doing so a somewhat different but more accurate results, also shown in Table 1, have been obtained. For surety, we have also crosschecked our results by

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Table 1
Comparison of the presently obtained results with those given in [1].

Variables	Results: values of variables					
	[1]	Present study				
		12 Significant digits	10 Significant digits	8 Significant digits	6 Significant digits	4 Significant digits
A (area of each effect, m^2)	591.9614	554.554347969	554.5543480	554.55435	554.554	554.6
W_{V0} (steam flow rate, kg/s)	3.8028	4.17412551448	4.174125514	4.1741255	4.17413	4.174
T_{V1} (steam temperature in 1st effect, °C)	111.0319	110.868966537	110.8689665	110.86897	110.869	110.9
T_{V2} (steam temperature in 2nd effect, °C)	94.3051	93.5868753063	93.58687531	93.586875	93.5869	93.59
T_{V3} (steam temperature in 3rd effect, °C)	80.5579	79.9956457261	79.99564573	79.995646	79.9956	80.
T_{V4} (steam temperature in 4th effect, °C)	69.0649	68.648633383	68.64863338	68.648633	68.6486	68.65
T_{V5} (steam temperature in 5th effect, °C)	60.2439	60.0369630578	60.03696306	60.036963	60.037	60.04
X_2 (product concentration in 2nd effect, kg/kg)	0.2883	0.282334151984	0.2823341520	0.28233415	0.282334	0.2823
X_3 (product concentration in 3rd effect, kg/kg)	0.2067	0.202219906183	0.2022199062	0.20221991	0.202220	0.2022
X_4 (product concentration in 4th effect, kg/kg)	0.1639	0.161094019449	0.1610940194	0.16109402	0.161094	0.1611
X_5 (product concentration in 5th effect, kg/kg)	0.1157	0.115222155159	0.1152221552	0.11522216	0.115222	0.1152
X_6 (product concentration in 6th effect, kg/kg)	0.1401	0.138603496848	0.1386034968	0.1386035	0.138603	0.1386

utilising the command 'FindRoot' in MATHEMATICA which is also a very efficient solver for nonlinear AEs. For both these solvers, the results given in [1] have been used as initial guesses and the same results, although different from those given in [1], were obtained. This is because the AEs resulting from such type of systems are highly nonlinear and in general difficult to solve, and for obtaining the results with desired accuracy one has to consider many significant digits in calculations [2,3]. Table 2 shows the residual values of Eqs. (1)–(12) after the substitution of presently obtained results as well as those given in [1]. This table not only indicates that the presently obtained results are more precise but it also shows the effect of number of significant digits present in the results on the residual values of AEs. It should also be noted that for equations to be satisfied if one sets their absolute values to be less than 10^{-6} [for $i = 1$–12, absolute value of Eq. (i) $\leqslant 10^{-6}$], then the minimum required precision in the variables is of 12 significant digits. The modelling equations of mixed feed sextuple evaporator system have been taken from [1,4] and are reproduced below. For other details, e.g. values of fixed variables and correlations used, the interested readers may refer to [1,4]. The values of overall heat transfer coefficients used in our program are as follows:

$U_1 = 1160$ W/m^2 K,
$U_2 = 1220$ W/m^2 K,
$U_3 = 1280$ W/m^2 K,
$U_4 = 1335$ W/m^2 K,

Table 2
Comparison of equations' values after substituting the results.

Eqn. No.	Eqns. values					
	[1]	For 12 significant digits	For 10 significant digits	For 8 significant digits	For 6 significant digits	For 4 significant digits
1	1286.51	-0.101027×10^{-6}	26.4669×10^{-6}	−0.0022791	−0.037921	−19.9896
2	−368.83	$-0.0414948 \times 10^{-6}$	1.01827×10^{-6}	−0.000259933	−0.00101976	−3.28389
3	422.522	0.111231×10^{-6}	-26.5846×10^{-6}	0.00252169	−0.00663812	18.1721
4	−200.541	0.0332211×10^{-6}	0.56758×10^{-6}	0.000787367	0.0303692	1.73307
5	749.523	0.033182×10^{-6}	1.2412×10^{-6}	0.000201589	0.0673671	−0.265771
6	−188.676	0.0370273×10^{-6}	-14.5483×10^{-6}	−0.00107939	−0.0398633	4.15827
7	545.877	0.0292093×10^{-6}	-6.58414×10^{-6}	0.00010922	−0.0303387	6.71965
8	−173.692	$-0.0723439 \times 10^{-6}$	8.94861×10^{-6}	0.00125635	−0.126811	−6.45305
9	383.117	$-0.0504997 \times 10^{-6}$	-7.53265×10^{-6}	0.000525505	−0.196557	−2.89563
10	−59.7996	0.23665×10^{-6}	-14.013×10^{-6}	−0.0028601	0.204158	11.4972
11	457.494	0.253201×10^{-6}	-14.5754×10^{-6}	−0.00211822	0.095946	12.4951
12	−22.3491	-0.419079×10^{-6}	48.9607×10^{-6}	0.00317074	0.0192679	−17.5751