Applied Mathematical Modelling 37 (2013) 6079–6096

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

An efficient statistically equivalent reduced method on stochastic model updating

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ARTICLE INFO

Article history:
Received 1 December 2011
Received in revised form 4 November 2012
Accepted 27 November 2012
Available online 2 January 2013

Keywords:
Stochastic response surface
Model updating
PCE
Uncertainty
Surrogate model

ABSTRACT

The demand for computational efficiency and reduced cost presents a big challenge for the development of more applicable and practical approaches in the field of uncertainty model updating. In this article, a computationally efficient approach, which is a combination of Stochastic Response Surface Method (SRSM) and Monte Carlo inverse error propagation, for stochastic model updating is developed based on a surrogate model. This stochastic surrogate model is determined using the Hermite polynomial chaos expansion and regression-based efficient collocation method. This paper addresses the critical issue of effectiveness and efficiency of the presented method. The efficiency of this method is demonstrated as a large number of computationally demanding full model simulations are no longer essential, and instead, the updating of parameter mean values and variances is implemented on the stochastic surrogate model expressed as an explicit mathematical expression. A three degree-of-freedom numerical model and a double-hat structure formed by a number of bolted joints are employed to illustrate the implementation of the method. Using the Monte Carlo-based method as the benchmark, the effectiveness and efficiency of the proposed method is verified.

1. Introduction

In the past several decades, model updating methods have been widely used for improving theoretical models for practical structures. In the traditional deterministic model updating, test data from a single structure is employed to update a single finite element (FE) model by minimizing the residual between predicted results and test data [1,2]. However, variability may exist in test data even for nominally identical structures. The source of various uncertainties mainly arises from two aspects, one from inevitable measurement noise and the other from geometric tolerances, manufacturing and assembling process, etc. Statistics based model updating methods were put forward by Collins et al. [3] in 1974. However, the application of these methods is confined to the treatment of only the variability in measurement noise on a single physical structure.

To consider the variability that exists not only from measurement noise, but also from the variation of structural parameters, joint connections and boundary supports, stochastic model updating has become a popular topic. Based on the statistical properties of experimental data, the statistical properties of parameters are optimized and adjusted following an uncertainty inverse propagation procedure. As far as the authors are concerned, Monte Carlo simulation (MCS)-based

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http://dx.doi.org/10.1016/j.apm.2012.11.026

approach and perturbation approach are the two kinds of most commonly employed uncertainty inverse propagation methods in stochastic model updating. In the MCS method, a large number of stochastic numerical simulation experiments with uncertain input parameters are necessary for the estimation of probability distribution of output parameters. The desired statistic characteristics of uncertain parameters are eventually evaluated from minimizing the residual of theoretical mean values and covariances with experimental results. Mares and Mottershead [4,5] used Monte Carlo inverse propagation and multivariate multiple regression stochastic model updating method to converge a set of analytical models with randomised updating parameters. The method was applied to the stochastic parameters updating of a set of nominally identical structures. But owing to the scatter of both updating parameters and experimental results, a large number of FE simulations are indispensable to the MCS-based stochastic model updating, and the associated costs can be excessively high. Govers and Link [6] used two independent recursive sets of equations for the updating of parameter mean values and covariance matrix simultaneously. The classical weighted least squares method was used for the adjustment of parameter mean values, and the adjustment of parameter covariance matrix was based on Frobenious norm. However, the application of the MCS in the stochastic finite element solution in each iteration step is still a heavy burden in terms of computational cost.

The perturbation method is an alternative method in stochastic model updating. In this method, a truncated Taylor series expansion around a linearization point is used to expand each term, in particular the first and the second moments in model updating equations [7]. Then the minimization of objective function results in the updating of parameter mean values and variances. Hua et al. [7] developed an improved perturbation method for the updating of parameter mean values and covariance matrix based on the adjustment of the first two moments of random structural parameters. However, the determination of second-order sensitivities causes expensive computation in this method. Khodaparast et al. [8] developed an efficient perturbation method by omitting the correlation between the updating parameters and measured response, and between the predicted response and measured response. However, all the existing perturbation methods are limited to small levels of uncertainties and sensitive to the original estimated parameter values.

In the field of stochastic model updating, computational efficiency is a big issue in the development of more applicable and practical approaches. The emergence of surrogate model conception provides a viable approach for the solution of complex computational problems. Among the existing surrogate model approaches, such as response surface method (RSM) [9,10], Kriging model [11–13], neural networks [14–17] and stochastic expansions method [18–23], the RSM has been commonly used for its ease of implementation and low computational cost. In the RSM, original FE-models are replaced by the approximate response surface models in the form of explicit polynomials [10]. However, as usually low order polynomials are adopted in the RSM for calculation efficiency, this method is not proper for highly non-linear issues. To overcome the limitation of low order polynomials in solving nonlinear problems in structural dynamics, the moving least squares response surface [24] and adaptive response surface [25] were put forward. Kriging model is used to estimate the values of unmeasured points based on the values of measured points, and the correlations between measured points and between measured points and unmeasured points, which provides a linear unbiased estimation for measured points by minimizing the variance of the prediction error [11]. With the combination of one deterministic term and one random term in the approximate function, the Kriging model is sufficiently flexible for global approximation and local approximation simultaneously. Therefore, the Kriging model is of higher accuracy in comparison with the RSM of the same order, but of lower efficiency as more terms are required in the Kriging approximation. Papadimitriou et al. [12] used the Kring method to optimize the sensor location in vibration experiment and Khodaparast et al. [13] developed an interval model updating method based on the Kriging predictor with irreducible uncertain measured data. Another kind of surrogate model-based updating strategy is based on the neural network algorithm. Two types of neural network procedures, such as multi-layer perceptron [15] and radial basis functions [16], have been employed to develop the neural networks framework for model updating. However, the intrinsic disadvantages behind the neural networks is that it is difficult to interpret the relationships between the response and the parameters since there is no direct relationship between them, therefore the neural networks are usually regarded as 'black-box' models. Stochastic expansions methods also constitute a good alternative of surrogate model with computational efficiency. The stochastic expansion is mainly used to better represent the characteristics of the stochastic system by a series of polynomials. Karhunen–Loeve (KL) expansions and polynomial chaos expansions (PCE) are two types of most frequently used stochastic expansions for their well behaved performance [18,19]. Although different orthogonal transforms of random variables are employed in the KL and the PCE, both are efficient in dimensional reduction of large numbers of random experiments. According to it the stochastic expansions interfere directly with the analysis of system, stochastic expansions are classified into two categories: the intrusive and the non-intrusive formulation procedures [18]. Spectral Stochastic FEM [19] and Stochastic Galerkin FEM [26] are two typical intrusive approaches, and probabilistic collocation method (PCM) [27] and stochastic response surface method (SRSM) [28] are two commonly employed non-intrusive approaches. Without the need to interfere with the finite element codes, non-intrusive approaches are more convenient for implementation and easy to integrate with commercial FE software. Adhikari and Friswell [21] used a KL expansion for the updating of spatially distributed parameters. In order to overcome the limitation of the PCM, Isukapalli [28] developed an improved PCM, which was called SRSM. In the SRSM method series expansions of random variables are used to approximate uncertain model outputs. Only a small number of model simulations are requisite in the calculation of the unknown coefficients in series expansions. In comparison with traditional methods in stochastic model updating, the implementation of SRSM reduces considerably the number of time-consuming full model calculations demanded for an appropriate estimate of the output statistics [27]. As an efficient and accurate statistically equivalent reduction approach, the SRSM has been used successfully