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Quasi-linear stochastic programming model based on expectation and variance and its application in transportation problem

Fachao Li [a,*], Chenxia Jin [a,*], Li Wang [b]

[a] School of Economics and Management, Hebei University of Science and Technology, Shijiazhuang 050018, China
[b] School of Traffic and Transportation, Beijing Jiaotong University, Beijing 100044, China

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ABSTRACT

The stochastic transportation problem involves in many areas such as production scheduling, facility location, resource allocation, logistics management. Constructing an operable solving method has important theoretical and practical value. In this paper, we first analyze the characteristic and deficiencies of the existing stochastic programming methods, such as higher computation complexity. We then give the concept of reliability coefficient and a quasi-linear processing pattern based on expectation and variance. We further analyze the relationship between reliability coefficient and reliability degree, also give the selecting strategy of reliability coefficient. Based on that, we establish a quasi-linear programming model for stochastic transportation problem, and we analyze its performance by a case-based example. The results indicate that this model has good interpretability and operability. It can effectively solve the transportation problem under complex stochastic environment or with incomplete information.

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1. Introduction

A stochastic programming is a very interesting well known problem in many fields such as production scheduling, facility location, resource allocation, logistics management. Establishing a solvable stochastic programming model has been widely concerned in academe and applications. At present, there are three generally acknowledged stochastic programming models: (1) Expected value model [1]. It uses mathematical expectation to describe random variables, then converts the stochastic programming into a deterministic one; (2) Chance-constrained programming model [2]. It converts stochastic constraints and objective functions into deterministic constraints and objective functions through some reliability principles; (3) Dependent-chance programming model [3,4]. It takes objective functions and constraints as events under random environment and gives the decision-making scheme by maximizing the chances of all the events happening.

Based on the above discussions, many scholars have made many beneficial discussions by combining with some concrete problems, and they have obtained good application results. Yang and Feng [5] established the expected value model, chance-constrained programming and dependent-chance programming of transportation problems; [6] gave a cross decomposition solution method for the stochastic transportation problem; [7,8] proposed fuzzy objective programming methods for stochastic transportation problems; [9] proposed bus dispatching chance-constrained programming model based on maximizing the business profit, for the uncertainty of running time and passenger demand; [10] presented a new chance constrained

* Corresponding authors. Tel.: +86 031181668359.
E-mail addresses: lifachao@tsinghua.org.cn (F. Li), jinchenxia2005@126.com (C. Jin).

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programming method for the optimal transmission system for several uncertain factors such as the locations and capacities of new power plants as well as demand growth; [11] used expected value model to consider single point O and much point D production synchronous scheduling problem of production network under random demand condition; [12] analyzed the distribution plan of uncertain demand and constructed a static secondary stochastic programming model, based on the chance-constrained model; [13] studied disaster prevention flood emergency logistics planning problems using chance-constrained programming; [14] introduced the concept of synthesis effect function, and gave the quasi-linear process pattern for stochastic constraints, further discussed the generalized expected value model of transportation problems and its solving.

Although the three models have been widely used in different fields, they all have some deficiencies: (1) Expected value model cannot effectively solve the decision-making problems under different risk, the quality of decision results cannot be guaranteed when the randomness is big; (2) Chance-constrained programming has the ability to control decision-making quality in advance, but the probability distributions of objective functions and constraints are often difficult to determine or precisely known, therefore, it is difficult to establish analytic methods under complex random environment; (3) Dependent-chance programming also involves the calculation problem of probability, thus the solving problem is also difficult to achieve; (4) When the distribution of random variable cannot be determined precisely, that is, the probability distribution function cannot be obtained, we cannot get the solution to the above three methods especially for the last two. Many scholars had many specific discussions and studies on these deficiencies. In terms of model solution, [15–19] constructed some solution methods through integrating random simulation and some intelligent algorithms (such as genetic algorithm, simulated annealing algorithm, particle swarm optimization algorithm), but many random simulation must involve lots of tests, so these methods are only suitable for small-scale stochastic programming problem. In terms of processing incomplete information, [20–25] introduced several methods for processing uncertainty and imprecision from different aspects, gave the corresponding information fusion strategies, and further presented satisfied decision-making schemes combining with appropriate algorithms. [26] put forward a multiple attribute decision optimization model with incomplete weight information, and gave a simple solution method; [27] designed a route planning system with incomplete information to plan a route by minimizing both danger and distance; [28] presented data analysis approaches of soft sets with incomplete information for the particularity of the value of mapping functions in soft sets, and this model can well represent incomplete data in soft sets, but it is worth noting that these discussions lack good structural features and generality.

From the above analysis, we can see that the existing methods still cannot effectively solve different stochastic programming problems in real life, and the common problems are stated as follows: (1) The complexity is high; (2) The precise distribution of random environment must be known.

Based on chance-constrained programming model, in this paper, we first propose a quasi-linear chance-constrained processing pattern based on expectation and variance, and also give the concept of reliability coefficient. We further analyze the relationship between reliability coefficient and reliability degree (probability), also give the selecting strategy of reliability coefficient. Based on that, we establish a quasi-linear programming model for stochastic transportation problem, and we analyze its performance by a case-based example.

The rest of the paper is structured as follows. Section 2 introduces the stochastic transportation problem. By the equivalent variation of the chance-constrained programming model, Section 3 proposes a quasi-linear chance-constrained processing pattern based on expectation and variance, gives the concept of reliability coefficient, and also analyzes the relationship between reliability coefficient and reliability degree, then gives the quasi-linear simplified strategy for the objective function and constraints. Moreover Section 3 establishes a quasi-linear programming model of stochastic transportation problem. Section 4 analyzes the characteristics and validity of the model combining with a case-based example. Some concluding remarks are drawn in Section 5.

In what follows, for the random variable ξ and event A on a probability space $(\Omega, B, \Pr)$, let $E(\xi)$ and $D(\xi)$ denote the mathematical expectation and variation of ξ, respectively, and $\Pr(A)$ the probability of A.

2. Formal description and characteristics of stochastic transportation problem

The transportation problem can be generally expressed: for a given goods, there are m sources $A_1, A_2, \ldots, A_m$ with outputs $a_1, a_2, \ldots, a_m$, respectively, and n destinations $B_1, B_2, \ldots, B_n$ with sales $b_1, b_2, \ldots, b_n$, respectively, and the freight of unit goods from source to destination is c_{ij}, try to determine the delivery scheme of the goods so that the total freight is the smallest.

Let x_{ij} ($i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$) be the quantity of products transported from source A_i to destination B_j, then the mathematical model of transportation problem can be expressed as:

$$\begin{cases} \min z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \\ \text{s.}t. \quad \sum_{j=1}^{n} x_{ij} \leqslant a_i, & i = 1, 2, \ldots, m, \\ \sum_{i=1}^{m} x_{ij} \geqslant b_j, & j = 1, 2, \ldots, n, \\ x_{ij} \geqslant 0, & i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n. \end{cases} \tag{1}$$

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