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The extended linear assignment method for multiple criteria decision analysis based on interval-valued intuitionistic fuzzy sets

Ting-Yu Chen *

Department of Industrial and Business Management, Graduate Institute of Business and Management, College of Management, Chang Gung University, 259, Wen-Hwa 1st Road, Kwei-Shan, Taoyuan 333, Taiwan

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ABSTRACT

The theory of interval-valued intuitionistic fuzzy sets is useful and beneficial for handling uncertainty and imprecision in multiple criteria decision analysis. In addition, the theory allows for convenient quantification of the equivocal nature of human subjective assessments. In this paper, by extending the traditional linear assignment method, we propose a useful method for solving multiple criteria evaluation problems in the interval-valued intuitionistic fuzzy context. A ranking procedure consisting of score functions, accuracy functions, membership uncertainty indices, and hesitation uncertainty indices is presented to determine a criterion-wise preference of the alternatives. An extended linear assignment model is then constructed using a modified weighted-rank frequency matrix to determine the priority order of various alternatives. The feasibility and applicability of the proposed method are illustrated with a multiple criteria decision-making problem involving the selection of a bridge construction method. Additionally, a comparative analysis with other methods, including the approach with weighted aggregation operators, the closeness coefficient-based method, and the auxiliary nonlinear programming models, has been conducted for solving the investment company selection problem to validate the effectiveness of the extended linear assignment method.

1. Introduction

Atanassov [1,2] introduced the concept of intuitionistic fuzzy sets. Later, Atanassov and Gargov [3] proposed the concept of interval-valued intuitionistic fuzzy (IVIF) sets, which are characterized by a membership function, a non-membership function, and a hesitancy function whose values are intervals. In the earlier stages of the development of IVIF sets, most research focused on basic operations and properties [4–8]. Based on the concept of IVIF sets, Xu [9] defined the notion of IVIF numbers and introduced some operations to be used with IVIF numbers. IVIF sets are more capable than traditional fuzzy sets at handling imprecise and uncertain information in practice. Additionally, IVIF set theory provides a useful and convenient method for dealing with equivocal properties in subjective judgments. IVIF sets have been applied productively in various fields, particularly in real situations involving multiple criteria decision analysis (MCDA), to facilitate coping with imprecise information.

With their sufficiency in the operations and properties of IVIF sets, several methods have been successfully developed to aggregate IVIF information. Xu and Chen [10] proposed several arithmetic aggregation operators, including the IVIF weighted arithmetic aggregation, ordered weighted aggregation, and hybrid aggregation operators, to synthesize IVIF preference

* Tel.: +886 3 2118800x5678; fax: +886 3 2118500.
E-mail address: tychen@mail.cgu.edu.tw

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information. Meanwhile, Xu [9] and [11] presented several operators in terms of geometrics, such as the IVIF weighted geometric, ordered weighted geometric, and hybrid geometric operators. Extending an induced ordered weighted geometric operator introduced by Xu and Da [12], Wei and Yi [13] constructed an induced IVIF ordered weighted geometric operator. Wei [14] developed an IVIF Choquet integral operator. Wei and Yi [15] used several IVIF aggregation operators to develop a multiple criteria decision-making method. Li [16] proposed a representation theorem of IVIF sets using the concept of level sets and developed the extension principle; moreover, the addition, subtraction, multiplication, and division operations in IVIF sets were defined based on the extension principle. Chen et al. [17] applied aggregation operators and IVIF preference relation matrices to develop a multiple criteria group decision-making method. Combining the IVIF geometric aggregation operator with the Choquet integral-based Hamming distance, Tan [18] solved group decision-making problems using an extension of the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method. Wei and Zhao [19] introduced induced intuitionistic fuzzy correlated averaging and geometric operators and extended these operators to the IVIF environment. Yu et al. [20] introduced some prioritized aggregation operators in the context of IVIF sets, specifically the prioritized weighted average and geometric operators, and applied them to group decision making. Meng et al. [21] used the Shapley function to develop an induced generalized IVIF hybrid Shapley averaging operator and proposed an approach to multiple criteria decision making.

Xu [9] introduced a score function and an accuracy function to measure IVIF numbers and developed a method for making comparisons between two IVIF numbers. Ye [22] proposed a novel accuracy function for IVIF sets and developed a decision-making method. In the context of IVIF numbers, Park et al. [23] investigated group decision-making problems, and they used score functions to calculate the score of each criterion value and constructed the score matrix of the collective IVIF decision matrix. Wang et al. [24] introduced a membership uncertainty index function. The difference between the levels of uncertainty in the membership and non-membership functions can be considered if both the score and the accuracy functions fail to distinguish between two IVIF numbers. Xu [25] built upon the concepts of the score and accuracy functions to develop a method via a distance measure for group decision making with IVIF matrices. Wei [26] introduced operational laws, score functions and accuracy functions of intuitionistic fuzzy sets and IVIF sets. He then developed new group decision-making methods using aggregation operators. Nayagam et al. [27] introduced a new novel accuracy function and presented an MCDA method based on IVIF sets. Based on the intuitionistic fuzzy weighted averaging operator, Mao et al. [28] utilized score functions and accuracy functions to rank the alternatives and applied the proposed method to solve group decision-making problems with IVIF information.

In addition to using aggregation operators, score functions, and accuracy functions, there are other techniques to handle MCDA problems based on IVIF sets, such as the linear programming model [24], TOPSIS [18], grey relational analysis [29], correlation coefficients [30], fuzzy cross entropy [31], and LINear programming technique for Multidimensional Analysis of Preference (LINMAP) [32]. Considering that imprecision and uncertainties may exist in subjective opinions and judgments, this paper represents multiple criteria evaluations and decision making in terms of IVIF sets. At present, the methods that employ aggregation operators, score functions, and accuracy functions are widely used approaches to the MCDA problem with IVIF sets as input data. For similar concepts, we intend to extend the linear assignment method introduced by Bernardo and Blin [33] to the IVIF decision environment.

Bernardo and Blin [33] utilized criterion-wise rankings and criterion weights to propound a linear assignment method of consumer choice among multiple brands. Several useful and valuable developments have enriched the linear assignment methodology in the field of multiple criteria evaluations, such as a fuzzy linear assignment approach [34]; a combined model using the Analytic Hierarchy Process (AHP), TOPSIS, and the linear assignment method [35]; a linear assignment method for ranking materials of engineering components [36]; a fuzzy linear assignment method for an initial order [37]; an interactive fuzzy linear assignment method [38]; and a nonlinear assignment-based model with IVIF sets [39]. On the basis of the main structure of the linear assignment methodology, we sought to develop an extended linear assignment method with IVIF settings to manage MCDA problems.

The purpose of this paper is to extend the linear assignment method to the decision environment indicated by IVIF sets for solving fuzzy MCDA problems. In comparison with the existing literature of the linear assignment methodology, the proposed extended linear assignment method has several significant advantages. First, the proposed method can handle the imprecise preference information and alternative ratings in a simple and effective manner. The classical linear assignment methods [33,35,36] cannot deal with imprecisions and quantify ambiguities in regard to subjective evaluations. Second, by utilizing IVIF theory, the proposed method can address MCDA problems in which decision makers find it difficult to assign precise membership values about criterion importance or alternative ratings with respect to criteria. Nevertheless, the existing fuzzy linear assignment methods [34,37,38] cannot handle uncertainty and imprecision in the degrees of membership. Third, the proposed method is simpler and easier to implement than the nonlinear assignment-based model. Although the nonlinear assignment-based model [39] can also address IVIF data, its computational cost is very high because of nonlinear constraints in the mathematical formulation. Fourth, the comparative analysis shows that the proposed method, relative to existing complicated MCDA methods, can yield credible solution results, while remaining far easier to use and understand.

This paper is organized as follows. Section 2 briefly reviews the concepts of IVIF sets. Section 3 first formulates an MCDA problem in which the evaluation of alternatives and criterion importance is expressed by IVIF sets. This section also develops an extended linear assignment method using the concepts of the score and accuracy functions, membership and hesitation uncertainty indices, and weighted-rank frequency matrices to determine the rank of the given alternatives. Section 4 demonstrates the feasibility and applicability of the proposed method by applying it to the MCDA problem of bridge