Applied Mathematical Modelling 38 (2014) 2141–2150

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

An efficient decision making approach in incomplete soft set

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ARTICLE INFO

Article history:
Received 8 October 2011
Received in revised form 3 September 2013
Accepted 8 October 2013
Available online 26 October 2013

Keywords:
Soft set
Incomplete information
Choice value
Weighted-average
Simplify probability

ABSTRACT

Zou et al. (2008) [21] presented weighted-average of all possible choice values approach of soft sets under incomplete information system in decision making. However, the approach is hard to understand and involves a great amount of computation. In order to simplify the approach, we present the simplified probability to directly instead of the incomplete information, and demonstrate the equivalence between the weighted-average of all possible choice values approach and the simplified probability approach. Finally, comparison results show that the proposed approach involves relatively less computation and is easier to implement and understand as compared with the weighted-average of all possible choice values approach.

1. Introduction

The vast amount of information generated by these applications can be valuable for data mining purposes. However, these data are often inherently associated with uncertainty environmental surveillance using large scale sensor networks, uncertainty is inherent in data due to various factors like incompleteness of data, limitations of equipment, and delay or loss in data transfer. These kinds of uncertainty have to be handled carefully, or else the mining results could be inaccurate or even incorrect.

The question naturally arises: how can small amounts of incomplete data be dealt with utilizing the available data? Statisticians approach this problem in a variety of ways: deleting incomplete entries; filling in incomplete entries based on the most similar complete entry ("hot deck imputation"); filling in incomplete entries with the sample mean ("mean substitution"); or using a learning algorithm or criterion (EM, max likelihood) to infer a missing entry [1].

Classical methods are not always successful, because the uncertainties appearing in these domains may be of various types. While probability theory, fuzzy sets [2], rough sets [3], and other mathematical tools are well-known and often useful approaches to describing uncertainty, each of these theories has its inherent difficulties as pointed out by Molodtsov [4].

As a general method for dealing with incomplete data, soft set theory has proven useful in many different fields such as decision making [5–19], medical science [20], data analysis [21,22], forecasting [23], simulation [24], evaluation of sound quality [25], rule mining [26]. By extending the classical soft sets, soft sets theory has combined with others theories such as fuzzy sets [27,28], rough sets [29–31], vague sets [32,33], interval-valued fuzzy sets [14,34], intuitionistic fuzzy sets [35–39], interval-valued intuitionistic fuzzy sets [40] and so on, and have been well studied and successfully applied in many fields.

Decision support analysis in the domain of a soft set is one of the most important applications. In the previous study researchers mainly discussed the complete data about decision making problem in soft set. While few research focus on data

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http://dx.doi.org/10.1016/j.apm.2013.10.009

analysis approaches under incomplete information. Zou et al. presented a data analysis approaches of soft sets under incomplete information, the decision value of an object with incomplete information is calculated by weighted-average of all possible choice values of the object [21]. An object-parameter method is proposed to predict unknown data in incomplete fuzzy soft sets by Deng and Wang [22]. While the weighted-average of all possible choice values approach is complex with great amount of calculations and difficult to understand. Even after adding parameters (or deleting parameters), or adding objects (or deleting objects), the all data must be recalculated again using the weighted-average of all possible choice values approach. For example, we have made decisions under incomplete information with weighted-average of all possible choice values approach, but some parameters (even only one parameter) may be added to the original parameter set (or be deleted from the original parameter set). The original decision results are ineffective and any data cannot be used for making new decisions after adding (or deleting) new parameters. We must recalculate and obtain new decisions, which waste more time. In this paper, we present the simplified probability approach, and demonstrate the equivalence between the weighted-average of all possible choice values approach and the simplified probability approach. Two approaches are compared from three aspects: complexity, adding (deleting) parameters, adding (deleting) objects. Some examples are provided to understand the significance of the simplified probability approach.

The rest of this paper is organized as follows. Section 2 reviews the basic definitions of soft set theory. Section 3 discusses the Weight-average of all possible choice value approach proposed in [21]. Section 4 the simplified approach is proposed and proof is given to demonstrate the equivalence of two approaches. Section 5 compare two approaches from three aspects: complexity, adding (deleting) parameters and adding (deleting) objects. Finally Section 6 presents the conclusion from our study.

2. Preliminaries

In the current section we will briefly recall some basic definitions for soft sets and give an example to illustrate soft sets.

Definition 2.1. A pair (F,E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U, i.e., $F{:}E \to P(U)$, where $P(U)$ is the power set of U.

The soft set is a parameterized family of subsets of the set U. Every set $F(\varepsilon)$, $\varepsilon \in E$, from this family may be considered as the set of ε-elements of the soft set (F,E), or as the ε-approximate elements of the soft set. As an illustration, some examples such as fuzzy sets and topological spaces were listed in [12]. The way of setting (or describing) any object in soft set theory differs in principle from the way it is used in classical mathematics. In classical mathematics, a mathematical model of an object is usually constructed which is too complicated to find the exact solution. Therefore the notion of approximate solution has been introduced in soft set theory, whose approach is opposite the classical mathematics.

To illustrate this idea, let us consider the following example.

Example 2.1. Let universe $U = \{h_1,h_2,h_3,h_4\}$ be a set of houses, asset of parameters $E = \{e_1,e_2,e_3,e_4\}$ be a set of status of houses which stand for the parameters "beautiful", "cheap", "in green surroundings", and "in good location" respectively. Consider the mapping F be a mapping of E into the set of all subsets of the set U. Now consider a soft set (F,E) that describes the "attractiveness of houses for purchase".

According to the data collected, the soft set (F,E) is given by
$\{F,E\} = \{(e_1,\{h_1,h_3,h_4\}), (e_2,\{h_1,h_2\}), (e_3,\{h_1,h_3\}), (e_4,\{h_2,h_3,h_4\})\}$
where $F(e_1) = \{h_1,h_3,h_4\}$, $F(e_2) = \{h_1,h_2\}$, $F(e_3) = \{h_1,h_3\}$, and $F(e_4) = \{h_2,h_3,h_4\}$.

In order to store a soft set in computer, a two-dimensional table is used to represent the soft set $\{F,E\}$. Table 1 is the tabular form of the soft set $\{F,E\}$. If $h_i \in F(e_j)$, then $h_{ij} = 1$, otherwise $h_{ij} = 0$, where h_{ij} are the entries (see Table 1).

Definition 2.2. Assume that we have a binary operation, denoted by $*$, for subsets of the set U, Let (F,A) and (G,B) be soft sets over U. Then, the operation $*$ for soft sets is defined in the following way: $(F,A) * (G,B) = (H,A \times B)$, where, $H(\alpha,\beta) = F(\alpha) * G(\beta)$, $\alpha \in A$, $\beta \in B$, and $A \times B$ is the Cartesian product of the sets A and B.

Suppose $U = \{h_1,h_2,\ldots,h_n\}$, $E = \{e_1,e_2,\ldots,e_m\}$, (F,E) is a soft set with tabular representation. Define $f_E(h_i) = \sum_j \{h_{ij}\}$, where h_{ij} are the entries in soft set table.

Table 1
The tabular representation of $\{F,E\}$.

U	e_1	e_2	e_3	e_4
h_1	1	1	1	0
h_2	0	1	0	1
h_3	1	0	1	1
h_4	1	0	0	1